



MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Core Course: Mathematics – I

AUMM141: Methods of Mathematics

(Common for **Regular** – 2016 Admn. and **Improvement** – 2015 Admn.)

Time: 3 Hours

Max. Marks: 80

SECTION – A

Answer ALL questions / problems in one or two sentences.

1. Write 1987_{10} to base 8.
2. State the Fundamental Theorem on Arithmetic.
3. Solve $5x \equiv 2 \pmod{7}$.
4. Find the natural domain of $f(x) = \cot x$.
5. Let $f(x) = x^5$ and $g(x) = 2x - 3$. Find $(f \circ g)(1)$.
6. Write the parametric equations of the cycloid in terms of the parameter θ .
7. State Intermediate – Value Theorem.
8. Given that $f(-2) = 3$ and $f'(-2) = -4$, find an equation for the tangent line to the graph of $y = f(x)$ at $x = -2$.
9. Find an equation for the ellipse with foci $(0, \pm 2)$ and major axis with endpoints $(0, \pm 4)$.
10. State the Reflection Property of Parabolas.

(10 × 1 = 10 Marks)

SECTION – B

Answer any EIGHT questions / problems, not exceeding a paragraph.

11. Using Euclid's algorithm, find the greatest common divisor of 17017 and 19210.
12. Prove that any two numbers a and b have a least common multiple.

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13. Find the least nonnegative residue of 2^{110} modulo 17.
14. Show that $2^{2n} - 3n - 1 \equiv 0 \pmod{9}$.
15. Classify the function $f(x) = \frac{x^5 - x}{1 + x^2}$ as even, odd or neither.
16. State the Squeezing Theorem.
17. Sketch the curve by eliminating the parameter from $x = 2\sin^2 t$ and $y = 3\cos^2 t$.
18. Find $\lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x^2 - 10x + 25}$.
19. Show that if f and g are continuous functions at c , then $f + g$ is also continuous at c .
20. Find the value of the constant A so that $y = A \sin 3t$ satisfies the equation $\frac{d^2 y}{dt^2} + 2y = 4 \sin 3t$.
21. Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm \frac{4}{3x}$.
22. Explain briefly of an application of parabola.

(8 × 2 = 16 Marks)

SECTION – C

Short essay type problems: Answer any SIX questions.

23. Prove by induction that $3^{2n} - 1$ is divisible by 8 for all $n \geq 0$.
24. If a divides bc , and a and b are coprime, then prove that a divides c .
25. Let $\mathbf{Z}$ be the set of all integers and ‘ m ’ be a fixed positive integer. Show that the relation ‘congruent modulo m ’ on $\mathbf{Z}$ is an equivalence relation.
26. Sketch the graph of $y = |x - 3| + 2$ by transforming the graph of $y = |x|$ appropriately.
27. Find $\lim_{x \rightarrow 0} f(x)$ where $f(x) = \frac{|x|}{x}$.
28. Show that the function $f(x) = \begin{cases} x^2 + x + 1, & x \leq 1 \\ 3x, & x > 1 \end{cases}$ is continuous at $x = 1$.
Determine whether the function f is differentiable at $x = 1$.
29. An aircraft is climbing at a 30° angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/h ?

30. The x and y axes are rotated through an angle of $\pi/4$ radians about the origin. Find an equation for the hyperbola $2xy = 9$ in the new coordinates.
31. Describe the graph of the equation $y^2 - 8x - 6y - 23 = 0$.

(6 × 4 = 24 Marks)

SECTION – D*Long essay type problems: Answer any TWO questions.*

32. a) Prove that the sequence of primes is endless.
 b) Prove that $\sqrt{2}$ is not rational.
 c) Prove the division theorem.
33. a) Find the first derivative of $y = \tan^4 \left(2 + \frac{(7-x)\sqrt{3x^2+5}}{x^3 + \sin x} \right)$.
 b) Find $\lim_{x \rightarrow +\infty} \sqrt{x^6 + 5x^3} - x^3$.
 c) Show that the equation $x^4 + 5x^3 + 5x - 1 = 0$ has at least two real solutions in the interval $[-6, 2]$.
34. a) Use implicit differentiation to find dy/dx for the Folium of Descartes $x^3 + y^3 = 3xy$.
 b) Find an equation for the tangent line to the Folium of Descartes at the point $(\frac{3}{2}, \frac{3}{2})$.
 c) At what point(s) in the first quadrant is the tangent line to the Folium of Descartes horizontal?
35. a) Define an ellipse.
 b) Derive the equation of an ellipse.
 c) Sketch the conic $16x^2 + 25y^2 = 400$. Include the foci, vertices and directrices in your sketch.

(2 × 15 = 30 Marks)
