MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :…………………**

First Semester B.Sc. Degree Examination, November 2014

First Degree Programme under CBCSS

Complementary Course: Mathematics – II (for Chemistry)

AUMM131.2b: Differentiation and Matrices

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. What is the vertical line test for a curve to be the graph of some function ?
2. Which is the natural domain of $x^2 - 3x + 2$?
   (i). $[-1, \infty)$ (ii). $(2, \infty)$ (iii). $(-\infty, 2]$ (iv). $(-\infty, \infty)$
3. Define absolute value of a real number.
4. What is the interpretation of slope m of a non – vertical line $y = (mx + b)$?
5. State Rolle's Theorem.
6. Define instantaneous rate of change of $y = f(x)$ with respect to x.
7. State Euler's theorem for homogenous function.
8. Define the rank of a matrix.
9. When do we say that a system of equations is consistent ?
10. Define equivalent matrices. Give an example.

(10 x 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Sketch the parametric curve $x = 2t - 1$, $y = t + 1$, $(1 \leq t \leq 2)$ and indicate its orientation.
12. Find the equation of the tangent line to the curve $y^2 = kx$ at (x_0, y_0).

13. A particle moves on a line away from its initial position so that after t hours it is $S = 3t^2 + t$ miles away from its initial position. Find the average velocity of particle over the interval [1, 3] and also the instantaneous velocity at $t = 1$.
14. Evaluate the following limits
    (i). $\lim_{y \to 6} \left| \frac{y+6}{y^2 - 36} \right|$
    (ii). $\lim_{x \to 0} \left| \frac{\sqrt{x+4} - 2}{x} \right|$
    (iii). $\lim_{x \to \infty} \left| \frac{\sqrt{3x^4 + x}}{x^2 - 8} \right|$
15. Find the Taylor series for $x \sin x$ about $x = \frac{\pi}{2}$.
16. Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$.
17. If $w = \sqrt{x^2 + 4y^2 - z^2}$. Find $\frac{\partial w}{\partial x}$.
18. Use the formula for the binomial series to obtain the Maclaurin's Series for $f(x) = \frac{1}{\sqrt{1+x}}$.
19. A point moves along the intersection of the elliptic paraboloid $z = x^2 + 3y^2$ and the plane $y = 1$. At what rate is z changing with x when the point is at (2, 1, 7) ?
20. What are the various elementary column transformations in matrices ?
21. When do we say that a matrix is in the normal form ?
22. Define the characteristic equation and eigen values of a matrix.

(8 x 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Find the Maclaurin series of for the function $\tan^{-1} x$.
24. (i). Describe the family of curves described by $x = a \cos(t + h)$, $y = b \sin(t + k)$, $0 \le t \le 2\pi$, where h and k are fixed but a and b can vary.
    (ii). Define a vertical and horizontal asymptote of the graph of the function $f(x)$.

(iii). Find the horizontal asymptote of $f(x) = \dfrac{3x+1}{x}$.

25. (i). One meter is about 6.214×10^{-4} miles. Find a formula $y = f(x)$ that expresses a length x in meters as a function of the same length y in miles.

(ii). Find a formula for the inverse of f.

(iii). In practical terms what does the formula $x = f^{-1}(y)\, x$ above tell you ?

26. (i). Compute $\dfrac{dz}{dt}$ where $z = 5x^2y^5 - 2x$, $x = t^2$ and $y = t^3 + 7$.

(ii). Let f be a differentiable function of one variable and let $z = f(x^2 + y^2)$. Show that $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y} = 0$.

27. Suppose that the temperature at a point (x,y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant walking on the plate traverses a circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant ?

28. For the matrix $A = \begin{bmatrix} -1 & 4 \\ 0 & 3 \end{bmatrix}$, find a matrix P such that $P^{-1}AP$ is diagonal.

29. (i). Define a homogenous and non – homogenous system of equations.

(ii). Show that the system of equations is consistent and solve them.

$x + 2y + z = 2$

$3x + y - 2z = 1$

$4x - 3y - z = 3$

$2x + 4y + 2z = 4$

30. Find the row reduced echelon form of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and determine its rank.

31. Find all eigen values and the eigen vectors corresponding to the largest eigen value of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

(6 x 4 = 24 Marks)

P.T.O.

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. (i). Determine the dimensions of a rectangular box open at the top having volume V and requiring the least amount of material for its construction.

(ii). Using Lagrange multipliers, find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point (1, 2, 2).

33. (i). Find three positive numbers whose sum is 48 and such that the sum of their squares is as small as possible.

(ii). The volume V of a right circular cone is given by $V = \frac{\pi}{24} d^2 \sqrt{4s^2 - d^2}$, where s is the slant height and d is the diameter of the base.

(a). Find a formula for the instantaneous rate of change of V with respect to d if s remains constant.

(b). Suppose that d has a constant value of 16cm, but s varies. Find the rate of change of V with respect to s when $s = 10$ cm.

34. (i). Using total differential, estimate the change in $Z = xy^2$ from its value at (0.5, 1.0) to its value at (0.503, 1.004). Compare the error in this estimate with the distance between the points (0.5, 1.0) and (0.503, 1.004).

(ii). Diagonalise the matrix $\begin{bmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{bmatrix}$.

35. (i). Test the consistency of the system and solve if consistent,

$2x_1 - 3x_2 = 1$

$-x_1 + 3x_2 = 0$

$x_1 - 4x_2 = 3.$

(ii). Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$.

(2 x 15 = 30 Marks)

∫*∫*∫*∫*∫*∫*∫*∫*∫*∫*∫*∫*∫*∫*∫*