MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :………………….**

Fifth Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Core Course: Mathematics – IV

AUMM541: Real Analysis I

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Fill in the blanks: If z and a are elements of R such that z + a = a, then z = ____________.
2. Define the ε – neighborhood of a real number a.
3. Determine the set A of all real numbers such that $2x + 3 \leq 6$.
4. Define a convergent sequence in R.
5. Give an example of an unbounded sequence that has a convergent subsequence.
6. Define Cauchy sequence in R.
7. Give example of an infinite set which has no cluster point.
8. State True/False: Absolute convergence of a series of real numbers implies convergence of that series.
9. Let $A \subseteq R$ and let $f: A \to R$, define left hand limit of f at $c \in R$.
10. Find $\lim_{x\to 2} \frac{x^2-4}{3x-6}$.

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. If $a, b \in R$ show that $a^2 + b^2 = 0$ iff $a = 0, b = 0$.
12. Let $J_n = (0, 1/n)$ for $n \in N$, prove that $\bigcap_{n=1}^{\infty} J_n = \emptyset$.

13. Show that there does not exist a rational number t such that $t^2 = 3$.
14. Show that the series $\sum_{n=1}^{\infty} \cos n$ is divergent.
15. Prove that a convergent sequence of real numbers is bounded.
16. Prove that the sequence $\{1/n\}$ is a Cauchy sequence.
17. Show that $\lim (b^n) = 0$ if $0 < b < 1$.
18. Discuss the convergence of the series $\sum_{n=1}^{\infty}(-1)^n = 1 - 1 + 1 - 1 + \cdots$
19. Show that the series $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$ converges. Find its sum.
20. Prove that $\lim_{x\to 0}\left(\frac{1}{x}\right)$ does not exist in R.
21. Find $\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}$.
22. Give examples of functions f and g such that f and g do not have limits at a point c, but such that both f + g and fg have limit at c.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems: Answer any **SIX** questions.*

23. State and prove the Archimedean property of real numbers.
24. State and prove Nested interval property.
25. Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then prove that the sequence $(\sqrt{x_n})$ of positive square roots converges and $\lim (\sqrt{x_n}) = \sqrt{x}$.
26. Define a contractive sequence and show that every contractive sequence is a Cauchy sequence.
27. Prove that the p – series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges when $p > 1$.
28. If (x_n) and (y_n) are sequences in R such that $\lim(x_n) = x$ and $\lim (y_n) = y$, show that $\lim (x_n y_n) = xy$.
29. Show that $\lim_{x\to\infty} \sin(1/x)$ does not exist in R.
30. Let $A \subseteq R$. Let f, g, h: $A \to R$ and let $c \in R$ be a cluster point of A. If $f(x) \leq g(x) \leq h(x)$ for all $x \in A$, $x \neq c$ and if $\lim_{x\to c} f = L = \lim_{x\to c} h$, then prove that $\lim_{x\to c} g = L$.
31. Show that $\lim_{x\to 0}\left(\frac{\cos x - 1}{x}\right) = 0$.

(6 × 4 = 24 Marks)

SECTION – D

Long essay type problems: Answer any ***TWO*** *questions.*

32. a) Let A and B be bounded non empty subsets of R and let A + B = {a + b: a ∈ A, b ∈ B}. Prove that sup (A + B) = sup A + sup B. **(5 Marks)**

    b) Prove that there exist a positive real number x such that $x^2 = 2$. **(10 Marks)**

33. a) State and prove Monotone Convergence Theorem. **(12 Marks)**

    b) Let X = (x_n) converges to x. Then prove that the sequence ($|x_n|$) of absolute values converges to |x|. **(3 Marks)**

34. a) State and prove Bolzano – Weirstrass theorem for convergent subsequence. **(10 Marks)**

    b) Prove that the series $\sum \frac{1}{n!}$ is convergent. **(5 Marks)**

35. a) State and prove sequential criterion for limits. **(10 Marks)**

    b) Show that $\lim_{x\to 0} \left(\frac{\sin x}{x}\right) = 1$. **(5 Marks)**

(2 × 15 = 30 Marks)

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