MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :…………………**

Third Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Complementary Course: Mathematics – III (for Physics)

AUMM331.2d: Differential Equations, Theory of Equations and Theory of Matrices

(*for 2014 Admissions – **Improvement** Only*)

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Solve the differential equation $xdy + ydx = 0$.
2. Write the standard form of a second order linear differential equation in two variables.
3. Write the characteristic equation of the differential equation, $(D^2 + 2D + 1)y = e^x$.
4. Define rank of a matrix.
5. If 1, 2, 3 are the eigen values of a matrix A then find the eigen values of A^t.
6. Write the echelon form of the matrix $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.
7. Form a quadratic equation with integer coefficients given that one of whose roots is $1 + i$.
8. Find a column basis for the row space of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
9. If 4, 2, 3 are the roots of the equation $ax^3 + bx + c = 0$. Find an equation whose roots are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.
10. Find the number of imaginary roots of the equation $x^3 + 2x + 3 = 0$.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Solve the differential equation $(1 + x^2)\, dy + (1 + y^2)\, dx = 0$.
12. Solve $y'' + 9y = e^x$.
13. Find the condition that the differential equation $(ax + by)\, dx + (kx + ly)\, dx = 0$ is exact.
14. Solve the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0,\ y(0) = 4,\ y'(0) = -5$.
15. Find the reduced form of the matrix $\begin{pmatrix} -3 & 3 & 3 \\ 0 & 2 & 2 \end{pmatrix}$.
16. Solve the system of equations $x + y + z = 0;\ x - y - z = 0$.
17. Test whether $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ is an eigen vector or not of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{pmatrix}$.
18. Solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, given that the roots are in arithmetic progression.
19. If α, β, γ are the roots of the equation $x^3 - 2x^2 + 3 = 0$, find the value of $\alpha^2 + \beta^2 + \gamma^2$.
20. Solve $2x^3 + x^2 - 7x - 6 = 0$, given that difference between two of its roots is 3.
21. Find a quartic equation with integer coefficients having $1 - i$ and $1 + \sqrt{2}$ as two of its roots.
22. If $A = \begin{bmatrix} 1 & 0 \\ 3 & 5 \end{bmatrix}$, then Show that, $A^2 + 6A + 5I = 0$, where I is the unit matrix of order 2.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Solve $(D^2 - 4D)y = 10\cos x + 5\sin x$.
24. Solve $(x^2 + xy)\frac{dy}{dx} = x^2 + y^2$.
25. Find a family of curves which are orthogonal to the family of curves $x^2 + (y - c)^2 = c^2$.
26. Using Newton – Raphson method, find a real root near 1.5 to four decimal places of the equation, $2x^3 - 7x^2 - x + 2 = 0$.
27. Find a positive root of the equation $x^2 - 3 = 0$ to two places of decimals using bisection method.

28. Solve $4x^4 - 4x^3 - 25x^2 + x + 6 = 0$, given that the difference between two of its roots is unity.
29. Test the consistency and solve: $2x + 3y + z = 6$; $x - y + z = 1$, $3x - y - z = 1$.
30. Find the eigen vectors of the matrix $\begin{pmatrix} -2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.
31. Solve $(D^2 - 3D + 2)\, y = x + \sin x$.

(6 × 4 = 24 Marks)

SECTION – D

Long essay type problems : Answer any ***TWO*** *questions.*

32. Show that the equation $e^{2x} = 25x - 20$ has two real roots and find the larger root correct to four significant figures.
33. Solve the following equations :
    i). $(x^2D^2 + xD - 4)\, y = 0$.
    ii). $(3x+2)^2 \dfrac{d^2y}{dx^2} + 3(3x+2)\dfrac{dy}{dx} - 36y = 3x^2 + 4x + 7$.
34. Solve:
    i). $x^2 y'' - xy' + y = \sin(ln\, x)$
    ii). $(D^2 - 3D + 2)y = e^{3x} \sin x$.
35. Diagonalise the matrix $\begin{pmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$, if possible.

(2 × 15 = 30 Marks)

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