



MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Complementary Course: Mathematics – I (for Chemistry and Physics)

AUMM131.2b / AUMM131.2d: Differentiation and Analytic Geometry

(for 2016 Admissions Only)

Time: 3 Hours

Max. Marks: 80

SECTION – A

Answer ALL questions / problems in one or two sentences.

1. Find an equation for the tangent line to the curve $y = 2x - 1$ at the point $(1,1)$ on the curve.
2. Find the slope intercept form of the equation of the line with slope 2 and y-intercept (-3) .
3. Find the slope of the curve $y = \sqrt{x}$ at $x = 9$.
4. Find $\frac{dy}{dx}$ if $y = \sqrt{x} \tan^3(\sqrt{x})$
5. Find $\frac{dy}{dx}$ if $x^{1/3} + y^{1/3} = a^{1/3}$
6. If $f(x, y) = \frac{x^2}{a^2} + \frac{y^2}{b^2}$, find f_{xy}
7. Find an equation of the parabola that is symmetric about the y-axis, has its vertex at the origin and passes through the point $(5,2)$.
8. Identify the conic: $16x^2 + 9y^2 - 64x - 54y + 1 = 0$
9. State Rolle's Theorem.
10. Convert the polar co-ordinates $\left(6, \frac{2\pi}{3}\right)$ into rectangular co-ordinates.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **EIGHT** questions / problems, not exceeding a paragraph.

11. Find $\frac{dz}{dt}$ in terms of t if $z = x^2 + 2xy + y^2$; $x = t + 1$; $y = t - 1$
12. Find the equation of a line passing through the point $(1, -2)$ and having slope $-3/4$
13. Find the angle of inclination of the line $y = \sqrt{3}x + 2$ to the nearest degree.
14. Use slopes to determine whether the points $(1,1)$, $(2, -1)$ and $(-2,1)$ lie on the same line.
15. If $x \neq 0$, show that $y = \frac{1}{x}$ satisfies the equation $x^3y'' + x^2y' - xy = 0$
16. Find the inflection points of $f(x) = \sin x$ on $[0, 2\pi]$
17. Locate the relative maxima and minima if any, of $f(x) = x^4 - 2x^2$
18. Graph the parametric curve $x = t - 3$, $y = t + 2$ by eliminating the parameter.
19. Evaluate: (i) $\lim_{(x,y) \rightarrow (-1,2)} \frac{xy}{x^2+y^2}$ (ii) $\lim_{(x,y) \rightarrow (-2,-1)} \frac{xy}{x^2+y^2}$
20. If $f(x, y) = y^2e^x + y$, find f_{xyy}
21. Express the equation $x^2 + y^2 = 1$ in polar co-ordinates.
22. Find the equation of the ellipse with foci $(0, \pm 2)$ and major axis with endpoints $(0, \pm 4)$.

(8 × 2 = 16 Marks)

SECTION – C

Short essay type problems : Answer any **SIX** questions.

23. State the vertical line test for functions. Use the same to check whether $x^2 + y^2 = 25$ defines y as a function of x ? Justify.
24. Find the domain and range of the function: $f(x) = \frac{x+1}{x-1}$
25. Evaluate: $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1}-1}$
26. Find the value of k if the following function is continuous everywhere:

$$f(x) = \begin{cases} 7x - 2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$$

27. Let $s(t) = 5t^2 - 22t$ be the position function of a particle moving along a coordinate line where s is in ft and t is in seconds. Find the average velocity of the particle during the time interval $1 \leq t \leq 3$.
28. Estimate the absolute maximum and minimum, if any, of $f(x) = x^{2/3}(20 - x)$
29. Verify Rolle's theorem for the function $f(x) = \frac{x}{2} - \sqrt{x}$ in the interval $[0, 4]$.
30. Sketch the graph of $r = \frac{6}{2 + \cos \theta}$ given in polar co-ordinates.
31. Find all values of x at which the tangent line to the curve $y = \frac{x+3}{x+2}$ is perpendicular to the line $y = x$.

(6 × 4 = 24 Marks)

SECTION – D*Long essay type problems : Answer any TWO questions.*

32. a) Evaluate: $\lim_{x \rightarrow 0} \left(x \sin \frac{1}{x} \right)$
- b) Find the average rate of change of $y = x^2 + 1$ with respect to x over the interval $[3, 5]$ and the instantaneous rate of change of y with respect to x when $x = -4$.
33. Identify and sketch the curve:

$$153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$$
34. a) Use implicit differentiation to find $\frac{dy}{dx}$ for the curve $x^3 + y^3 = 3xy$. Find the equation for the tangent line at $\left(\frac{3}{2}, \frac{3}{2}\right)$. At what points will be the tangent horizontal ?
- b) Find the slope of the surface $z = x^2y + 5y^3$ in the x -direction at $(1, -2)$.
35. An open box is to be made from a 3 inch by 8 inch piece of cardboard by cutting squares of equal size from four corners and folding up the sides. What size square should be cut to obtain a box with largest possible volume ?

(2 × 15 = 30 Marks)

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