MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :…………………**

Third Semester B.Sc. Degree Examination, November 2016
First Degree Programme under CBCSS
Complementary Course: Statistics – III (for Mathematics)
AUST331.2c: Probability Distributions and Theory of Estimation

(*for 2014 Admissions – **Improvement** Only*)

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Define a continuous uniform distribution.
2. If X follows a binomial distribution (6, 0.3), what is the distribution of 6 – X ?
3. Find out the 7th central moment of N(μ, σ).
4. Define a hypergeometric distribution.
5. Define convergence in probability.
6. Distinguish between statistic and parameter.
7. Define the term unbiasedness property in estimation.
8. Define the term point estimation.
9. Name the properties of maximum likelihood estimators.
10. What is the distribution of sum of two independent exponential random variables with parameter θ ?

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Find the moment generating function of X if X follows the uniform distribution and takes values 1, 2, 3, 4. Compute its mean from m.g.f.
12. Find the mean for the Geometric distribution.

13. Define an exponential distribution. Compute its moment generating function.
14. Prove that the mode of the distribution $P(X) = (1/2)^x$, x = 1, 2, 3, ... is 1.
15. Give example of an estimate which is not unbiased but is consistent.
16. Find the confidence interval for μ when σ is unknown.
17. X follows Binomial distribution with parameters n = 6 and p.
    If 9P(X = 4) = P(X = 2), find p.
18. Define central limit theorem with the assumptions made about random variables.
19. Define the term method of moments with an example.
20. Define Beta distribution of the first kind. Obtain its mean and variance.
21. Define a normal distribution with mean μ and variance σ^2. Compute its median ?
22. State Bernoulli's weak law of large numbers.

(8 × 2 = 16 Marks)

SECTION – C

Short essay type problems : Answer any ***SIX*** *questions.*

23. If X and Y are independent Poisson random variables with parameters μ and λ. Find the conditional distribution of X given X + Y.
24. If μ_r denotes the r^{th} central moments of a binomial distribution, derive the recurrence relation for the central moments.
25. State and prove additive property of Gamma distribution.
26. For a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and S.D.
27. Define a Student's '*t*' distribution. Write down two statistics which follow '*t*' distribution.
28. An unbiased coin is tossed 100 times. Show that the probability that the number of heads will be between 30 and 70 is greater than 0.93, using Tchebycheff's inequality.
29. A random sample of size *n* was taken from a population with mean μ and standard deviation σ. Find the distribution of the sample mean for larger values of *n*.
30. Find the confidence interval for the variance of a normal population.
31. Explain Cramer – Rao inequality. Show that for a normal distribution $N(\mu, \sigma)$ with σ known, sample mean is the minimum variance estimator of μ.

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). Obtain the Poisson distribution as a limiting form of the Binomial distribution. **(7 Marks)**

    ii). State and prove Tchebycheff's inequality. **(8 Marks)**

33. Five dice were thrown 96 times and the number of times atleast one die showed an even number is given below. Fit a Binomial Distribution and find the theoretical frequencies of the given data.

No.of dice showing even number x :	0	1	2	3	4	5
Frequency f :	3	8	24	35	19	7

(15 Marks)

34. i). Establish the relation between Normal, Standard Normal, student's *t*, chi–square and F– distribution. **(8 Marks)**

    ii). A random sample is taken from a normal population with mean 30 and SD 4. How large a sample should be taken if the sample mean is to lie between 25 and 35 with probability 0.98 ? **(7 Marks)**

35. i). Define a Chi – square distribution with '*n*' degrees of freedom. Compute its mean, variance and m.g.f . **(10 Marks)**

    ii). A random sample of size 10 taken from a normal population N (μ, σ) has SD 4. Find 'a' and 'b' such that $P(a \leq \sigma^2 \leq b) = 0.95$? **(5 Marks)**

(2 × 15 = 30 Marks)

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