MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :…………………..**

First Semester B.Sc. Degree Examination, November 2015
First Degree Programme under CBCSS
Complementary Course: Mathematics – I (for Physics)
AUMM131.2d: Differentiation and Analytic Geometry
(*for 2015 Admissions Only*)

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Find the slope of the inclined plane making an angle 45° with the base.
2. Find the natural domain of $f(x) = \tan x$.
3. Find $\lim_{x\to 5}(x^2 - 4x + 3)$.
4. Find the average rate of change of $y = x^3 + 1$ with respect to x over the interval [3, 5].
5. Define absolute minimum of a function in an interval I.
6. State Rolle's theorem.
7. Evaluate $\lim_{(x,y)\to(1,4)}\left[5x^3y^2 - 9\right]$.
8. State the extreme value theorem.
9. Find the rectangular coordinates of the point P whose polar coordinates are $(6, \frac{2\pi}{3})$.
10. State the Reflection Property of Ellipses.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Suppose that a car moves in the positive direction of an s – axis in such a way that its velocity 'v' increases at a constant rate of 2 ft/s^2. Assuming that the velocity of the car is 88 ft/s at time $t = 0$, find the equation for 'v' as a function of t.
12. Define continuity of a function f at x = c.
13. Compute $\lim_{x \to 1} \frac{\sin(\pi x)}{x-1}$.
14. Find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$.
15. Locate the relative extrema of $f(x) = x^3 - 3x^2 + 3x - 1$, if any.
16. Find the slope of the curve $y = x^2 + 1$ at the point (2, 5) and use it to find the equation of the tangent at $x = 2$.
17. Find the velocity and acceleration of a particle which moves on a parabola $s(t) = 16t^2 - 29t + 6$ at $t = 3$.
18. Find $\frac{d}{dx}$ [ln (tan hx)].
19. Determine $f_x(1, 3)$ for the function $f(x, y) = 2x^3y^2 + 2y + 4x$.
20. Show that when f is differentiable, a function of the form $z = f(xy)$ satisfies the equation $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$.
21. Find the equation of the parabola with focus (0, –3) and directrix $y = 3$
22. Sketch the graph of the curve $r = 1$ in polar coordinates.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Find the graph of the parametric equations.
    $x = \cos t$ and $y = \sin t$, $(0 \le t \le \pi)$.
24. Show that $|x|$ is continuous everywhere.

25. Prove that $\lim_{x\to 0}\frac{\sin x}{x}=1$.

26. A liquid form of penicillin manufactured by a pharmaceutical firm is sold in bulk at a price of Rs.200/– per unit. If the total production cost (in rupees) for x units is $C(x) = 5000000 + 80x + 0.003x^2$ and if the production capacity of the firm is at most 30,000 units in a specified time, how many units of penicillin must be manufactured and sold in that time to maximize the profit ?

27. Find the interval [a, b] on which $f(x) = x^4 + x^3 - x^2 + x - 2$ satisfies the Rolle's theorem.

28. Find $\frac{d}{dx}$ [ln $|x|$].

29. Let $u = (x^2 + y^2 + z^2)^{-1/2}$. Show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$.

30. Describe the graph of the equation $x^2 - y^2 - 4x + 8y - 21 = 0$.

31. Find the equation of the ellipse with foci (0, ±2) and major axis with end points (0, ±4).

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). Sketch the graph of $y = 2 - \frac{1}{x+1}$ by transforming the graph of $y = \frac{1}{x}$ appropriately.

ii). Evaluate the following limits.

(a). $\lim_{x\to 1}\frac{x^3-1}{x-1}$. (b). $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$

33. i). Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

ii). Show that $\sinh^{-1} x = \ln(x + \sqrt{(x^2+1)})$.

34. i). Let $f(x, y) = x^2y + 5y^3$. Find the slope of the surface $z = f(x, y)$ in the x – direction at the point (1, –2).

ii). Verify Euler's theorem for the function $f(x, y) = (x^2 + xy + y^2)^{-1}$.

iii). Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3, and requiring the least amount of material for its construction.

35. i). Describe the graph of the function $x^2 - y^2 - 4x + 8y - 21 = 0$.

ii). Find the new coordinates of the point (2, 4) if the coordinate axes are rotated through an angle of $\theta = 30^\circ$.

iii). For the conic $r = \dfrac{3}{2 - 2\cos\theta}$, find the eccentricity and distance from the pole to the directrix and sketch the graph in polar coordinates.

(2 × 15 = 30 Marks)

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