## MAR IVANIOS COLLEGE (AUTONOMOUS) THIRUVANANTHAPURAM

Reg. No. :
Name :
Fifth Semester B.Sc. Degree Examination, November 2016
First Degree Programme under CBCSS
Open Course: Mathematics - I
AUMM581: Operations Research
Time: $\mathbf{3}$ Hours
Max. Marks: 80

## SECTION - A

Answer ALL questions / problems in one or two sentences.

1. What is a Linear Programming Problem ?
2. Identify the objective function of the L.P.P: Maximize $Z=8 x_{1}+6 x_{2}$ subject to $4 x_{1}+2 x_{2} \leq 60,2 x_{1}+4 x_{2} \leq 48, x_{1}, x_{2} \geq 0$.
3. Is $x_{1}=2, x_{2}=1$, a feasible solution of the L.P.P: Minimize $Z=x_{1}+2 x_{2}$ subject to $2 x_{1}-x_{2} \geq 2, \quad x_{1}+3 x_{2} \leq 8 ; \quad x_{1}, x_{2} \geq 0$ ?
4. Name any one advantage of simplex method over graphical method in solving a linear programming problem.
5. Define a transportation problem.
6. Write a necessary and sufficient condition for the existence of a feasible solution to an $m \times n$ transportation problem.
7. Write the name of a method used to find the optimal solution of a transportation problem.
8. Name a method for solving assignment problem.
9. Define an event.
10. Name two techniques for Project management.

## SECTION - B

Answer any EIGHT questions / problems, not exceeding a paragraph.
11. A house wife wishes to mix two types of food $F_{1}$ and $F_{2}$ in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food $\mathrm{F}_{1}$ costs Rs. $60 / \mathrm{Kg}$ and Food $\mathrm{F}_{2}$ costs Rs. $80 / \mathrm{Kg}$. Food $\mathrm{F}_{1}$ contains 3 units $/ \mathrm{Kg}$ of vitamin A and 5 units $/ \mathrm{Kg}$ of vitamin B while Food $\mathrm{F}_{2}$ contains 4 units $/ \mathrm{Kg}$ of vitamin A and 2 units $/ \mathrm{Kg}$ of vitamin B. Formulate this problem as a linear programming model to minimize the cost of mixtures.
12. What are the basic components of a linear programming problem?
13. Define Slack and Surplus variables ?
14. Convert the following constraints into equations:

$$
2 x_{1}+x_{2}+3 x_{3} \leq 2 ; \quad x_{1}-4 x_{2}+x_{3} \geq 3
$$

15. Write the following LPP in the standard form:

Maximize $Z=3 x_{1}+2 x_{2}+10 x_{3}$

$$
\begin{array}{ll}
\text { Subject to } & x_{1}+x_{2}+4 x_{3} \geq 4 \\
& x_{1}-x_{2}+2 x_{3} \leq 6 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

16. Represent the following LPP in matrix - vector notation

$$
\begin{aligned}
\text { Maximize } Z=2 & x_{1}+4 x_{2}-3 x_{3}+x_{4} \\
\text { Subject to } & x_{1}+2 x_{2}+x_{3}+5 x_{4}=10 \\
& x_{2}-2 x_{3}+x_{4}=7 \\
& x_{1}+7 x_{2}+3 x_{3}+x_{4}=2 \\
& x_{1}, x_{2}, x_{3}, x_{4} \geq 0
\end{aligned}
$$

17. Write the Linear Programming formulation of a Transportation Problem.
18. How does the problem of degeneracy arise in a transportation problem?
19. Use North - West Corner method to find an initial basic feasible solution to the following transportation problem:

|  | $\mathrm{W}_{1}$ | $\mathrm{~W}_{2}$ | $\mathrm{~W}_{3}$ | Availability |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{F}_{1}$ | 16 | 20 | 12 | 200 |
| $\mathrm{~F}_{2}$ | 14 | 8 | 18 | 160 |
| $\mathrm{~F}_{3}$ | 26 | 24 | 16 | 90 |
| Demand | 180 | 120 | 150 |  |

20. What is an assignment problem ? Explain.
21. Give any two applications of CPM or PERT.
22. What are the three time considerations in PERT ?
( $8 \times 2=16$ Marks )

## SECTION - C

Short essay type problems: Answer any SIX questions.
23. Use graphical method to solve the following LP problem:

Minimize $Z=3 x_{1}+2 x_{2}$
Subject to $5 x_{1}+x_{2} \geq 10$

$$
x_{1}+x_{2} \geq 6
$$

$$
x_{1}+4 x_{2} \geq 12
$$

$$
x_{1}, x_{2} \geq 0
$$

24. Solve the following LPP using Simplex Method:

Maximize $Z=7 x_{1}+5 x_{2}$

$$
\begin{array}{ll}
\text { Subject to } & x_{1}+2 x_{2} \leq 6 \\
& 4 x_{1}+3 x_{2} \leq 12 \\
& x_{1}, x_{2} \geq 0
\end{array}
$$

25. Determine an initial basic feasible solution to the following transportation problem using North - West Corner Rule.

|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ | Availability |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | 6 | 4 | 1 | 4 | 14 |
| $\mathrm{O}_{2}$ | 8 | 9 | 2 | 7 | 16 |
| $\mathrm{O}_{3}$ | 4 | 3 | 6 | 2 | 5 |
| Demand | 6 | 10 | 15 | 4 |  |

26. Use Vogel's Method to find an initial basic feasible solution to the following transportation problem:

|  | D | E | F | G | Availability |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 11 | 13 | 17 | 14 | 250 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| Demand | 200 | 225 | 275 | 250 |  |

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27. Find an initial basic feasible solution for the following transportation problem using North - West Corner Rule:

|  | $\mathrm{M}_{1}$ | $\mathrm{M}_{2}$ | $\mathrm{M}_{3}$ | $\mathrm{M}_{4}$ | Availability |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{W}_{1}$ | 4 | 6 | 8 | 13 | 500 |
| $\mathrm{~W}_{2}$ | 13 | 11 | 10 | 8 | 700 |
| $\mathrm{~W}_{3}$ | 14 | 4 | 10 | 13 | 300 |
| Demand | 250 | 350 | 650 | 250 |  |

28. Obtain the optimal assignment of four jobs and four machines when the cost of assignment is given by the following table:

|  | $\mathbf{J}_{1}$ | $\mathbf{J}_{2}$ | $\mathbf{J}_{3}$ | $\mathbf{J}_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{M}_{1}$ | 12 | 30 | 21 | 15 |
| $\mathbf{M}_{2}$ | 18 | 33 | 9 | 31 |
| $\mathrm{M}_{3}$ | 44 | 25 | 21 | 21 |
| $\mathrm{M}_{4}$ | 14 | 30 | 28 | 14 |

29. Use Hungarian method to solve the following assignment problem:

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 9 | 7 | 8 |
| 2 | 5 | 8 | 7 | 7 |
| 3 | 5 | 4 | 6 | 5 |
| 4 | 2 | 3 | 4 | 5 |

30. A batch of four jobs can be assigned to five different machines. The setup time for each job on various machines is given by the following table:

|  | $\mathbf{M}_{1}$ | $\mathbf{M}_{2}$ | $\mathbf{M}_{3}$ | $\mathbf{M}_{4}$ | $\mathbf{M}_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{J}_{1}$ | 10 | 11 | 4 | 2 | 8 |
| $\mathrm{~J}_{2}$ | 7 | 11 | 10 | 14 | 12 |
| $\mathrm{~J}_{3}$ | 5 | 6 | 9 | 12 | 14 |
| $\mathrm{~J}_{4}$ | 13 | 15 | 11 | 10 | 7 |

Find an optimal assignment of jobs to machines which will minimize the total setup time.
31. A project with 5 jobs and with the following job sequence is given. Draw a project network. Write all the paths of the project and hence identify its critical path:

| Activity | A | B | C | D | E |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Sequence | $1-2$ | $1-3$ | $2-4$ | $3-4$ | $4-5$ |
| Completion time (days) | 3 | 1 | 4 | 2 | 5 |

( $6 \times 4=24$ Marks)

## SECTION - D

Long essay type problems: Answer any TWO questions.
32. Solve the following LPP:

$$
\begin{array}{lr}
\text { Maximize } \\
\text { Subject to } & Z=x_{1}+2 x_{2} \\
& x_{1}+2 x_{2} \leq 4 \\
& x_{1}+7 x_{2} \leq 14 \\
& x_{1}-x_{2} \leq 1 \\
& x_{1}, x_{2} \geq 0
\end{array}
$$

33. Find an initial basic feasible solution for the following transportation problem using any two different methods and critically evaluate the methods:

|  | $\mathbf{M}_{1}$ | $\mathbf{M}_{2}$ | $\mathbf{M}_{3}$ | $\mathbf{M}_{4}$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{W}_{1}$ | 2 | 2 | 2 | 1 | 3 |
| $\mathrm{~W}_{2}$ | 10 | 8 | 5 | 4 | 7 |
| $\mathrm{~W}_{3}$ | 7 | 6 | 6 | 8 | 5 |
| Demand | 4 | 3 | 4 | 4 |  |

34. The following table gives the activities in a project and other relevant information.
i) Draw the network of the project.
ii) Find the forward pass and backward pass.
iii) Find the critical path and total duration of the project.

| Activity | Sequence | Duration |
| :--- | :---: | :--- |
| A | $1-2$ | 10 |
| B | $2-3$ | 12 |
| C | $2-4$ | 5 |
| D | $3-4$ | 6 |
| E | $4-5$ | 3 |

35. Consider a project consisting of 7 jobs: $\mathrm{A}, \mathrm{B}, \ldots, \mathrm{G}$ with the following job sequence and time estimates:

| Job | Sequence | Optimistic <br> time (a) | Most probable <br> time (m) | Pessimistic <br> time (b) |
| :--- | :---: | :---: | :---: | :---: |
| A | $1-2$ | 2 | 5 | 8 |
| B | $1-3$ | 6 | 9 | 12 |
| C | $2-3$ | 3 | 6 | 9 |
| D | $3-5$ | 1 | 4 | 7 |

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| E | $3-4$ | 8 | 8 | 8 |
| :---: | :---: | :---: | :---: | :---: |
| F | $4-5$ | 5 | 14 | 17 |
| G | $5-6$ | 3 | 12 | 21 |

i). Draw the network
ii). Find the average time and variance of each job
( $2 \times 15=30$ Marks $)$
$\int * \int * \int * \int * \int * \int * \int * \int * \int * \int * \int * \int * \int * \int * \int *$

