MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :………………….**

Third Semester B.Sc. Degree Examination, November 2015
First Degree Programme under CBCSS
Complementary Course: Mathematics – III (for Chemistry)
AUMM331.2b: Vector Analysis and Theory of Equations

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Find $\lim_{t\to 2}(ti - 3j + t^2k)$.
2. Let $f(x, y, z) = x^2 + y^2 - z$. Find an equation of the level surface that passes through the point (1, – 2, 0).
3. If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$, find gradient of ϕ at (1, –1, 2).
4. Evaluate the integral $\int_C (3x + 2y)dx + (2x - y)dy$ along the line segment C from (0, 0) to (1, 1).
5. State Stoke's theorem.
6. Determine whether $\vec{F}(x, y) = 3y^2 i + 6xyj$ is a conservative vector field.
7. Let $\vec{F}(x, y)$ be a conservative vector field on a region D and C a smooth closed curve in D. What is the value of $\int_C \vec{F}.d\vec{r}$?
8. State the fundamental theorem of algebra.
9. What is the sum of the four roots of the equation $3x^4 + 2x^3 + 5x^2 + 7x - 4 = 0$?
10. How many real roots does the equation $2x^2 + 2 = 0$ have ?

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Find the unit tangent vector to the graph of $\vec{r}(t) = 4\cos t\, i + 4\sin t + 4\sin t\; j + t\, k$ at the point where $t = \frac{\pi}{2}$.
12. Find the velocity and acceleration of a particle moving along the curve $x = t,\ y = \frac{1}{2}t^2,\ z = \frac{1}{3}t^3$ at time $t = 1$.
13. Find the constants a, b, c such that $\vec{F}(x, y, z) = (x + 2y + az)i + (bx - 3y - z)\,j + (4x + cy + 2z)k$ is irrotational.
14. Prove that $curl(\nabla\varphi) = \vec{0}$, where φ is a scalar function.
15. Find the work done by the force field $\vec{F}(x, y) = x^3 y\, i + (x - y)\, j$ on a particle that moves along the parabola $y = x^2$ from (–2, 4) to (1, 1).
16. Find the value of $\int_{(1,2)}^{(4,0)} (3y\, dx + 3x\, dy)$.
17. Find a unit normal vector to $r(t) = t\,i + t^2 j$ at the point where $t = 1$.
18. Find the divergence of the vector field $\vec{F}(x, y, z) = e^{xy} i - \cos y\; j + \sin^2 z\, k$.
19. Solve the equation $2x^3 + 3x^2 - 11x - 6 = 0$, given that the three roots form an arithmetic sequence.
20. Show that $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.
21. State Descarte's rule of signs.
22. Solve the equation $x^3 - 5x^2 - 8x + 12 = 0$, given that the sum of two of the roots is 7.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Find the curvature of the curve $\vec{r}(t) = e^t i + e^{-t} j + t\, k$ at the point where $t = 0$.
24. Find the directional derivative of $f(x, y, z) = x^3 z - yx^2 + z^2$ at the point (2, –1, 1) in the direction of the vector $\vec{a} = 3i - j + 2k$.
25. If $\vec{r} = xi + y\; j + z\, k$ and $r = |\vec{r}|$, then show that gradient of $\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$.

26. Using Green's theorem, find the work done by the force field $\vec{F}(x, y) = (x^2 - y^2)i + x\, j,$ on a particle that travel once around the circle $x^2 + y^2 = 9$ in the counterclockwise direction.
27. Use the Divergence theorem to find the outward flux of $\vec{F}(x, y, z) = (x^2 + y)i + z^2 j + (e^y - z)k,$ across the surface of the rectangular solid bounded by the coordinate planes and the planes $x = 3$, $y = 1$ and $z = 2$.
28. Evaluate $\iiint_G (3x^2 + 3y^2 + 2z)dV$ where G is the region enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.
29. Find a real root of the equation $x^3 - 2x - 5 = 0$ using bisection method.
30. Solve the equation $x^3 - 10x^2 + 8x + 64 = 0,$ given that the product of two of the roots is the negative of the third.
31. Explain the Newton – Raphson method of finding a solution to the equation $f(x) = 0$.

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. If $\vec{F}(x, y, z) = xy^2 i - 3yz\, j + xy\, k,$ find $\nabla \times (\nabla \times \vec{F})$ and $\nabla . (\nabla \times \vec{F})$.
33. Evaluate the surface integral $\iint_\sigma y^2 z^2\, dS$ where σ is the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the planes $z = 1$ and $z = 2$.
34. The equation $x^3 - 3x - 4 = 0$ has a root between $x = 1$ and $x = 3$. Using Newton – Raphson method, evaluate this root to four significant digits.
35. Verify Stoke's theorem for the vector field $\vec{F}(x, y, z) = 2z\, i + 3x\, j + 5y\, k,$ taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy – plane.

(2 × 15 = 30 Marks)

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