# MAR IVANIOS COLLEGE (AUTONOMOUS)
# THIRUVANANTHAPURAM

**Reg. No. :……** **Name :…….**

**Second Semester B.A. Degree Examination, June 2016**
**First Degree Programme under CBCSS**
**Complementary Course: Mathematics – II (for Economics)**
**AUMM231.1a: Mathematics for Economics – II**

Time: **3** Hours Max. Marks: **80**

## SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Find the two first order partial derivatives of $y = xt^3 + 3t^2 + x^2t$.
2. State Euler's theorem for homogeneous function.
3. If the demand is given by the linear function $p = a - bx$, find the total revenue and the marginal revenue functions.
4. Define optimum feasible solution to an LPP.
5. Determine whether $y = x(x-2)$ is decreasing at $x = 2$.
6. Find the minimum point of $y = x^2$.
7. Define a convex set.
8. Find the slope of the tangent at (3, 3) to the curve $xy = 1000$.
9. If $x = 3t^2$, $y = 2t$, find $\frac{dy}{dx}$ at $t = 1$.
10. Find the points of inflexion of the curve $y = x^4 - 4x^3 + 16x$.

**(10 × 1 = 10 Marks)**

## SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Find $\frac{du}{dt}$ where $u = x^2 + y^2$, $x = at^2$, $y = 2at$, $a$ is an arbitrary constant.

12. Define monotonic decreasing function. Also determine whether $y = x^2$ is monotonic decreasing or not.
13. Define inflexion point. Check whether $x = 0$ is an inflexion point of $x^4$
14. Suppose a firm sells 20,000 units when the price is \$16, but sells 30,000 units when the price falls to \$14. Find the price elasticity of demand over this range of prices. State whether demand is elastic or inelastic over this range.
15. State Young's theorem for partial derivatives and verify it for $u = e^{(x^2 + y^2)}$
16. Find the relative minimum and relative maximum value of the function, $z = x^2 + y^2 + 6x + 12$
17. If $z = \sqrt{x^2 + y^2}$, $x^3 + y^3 + 3axy = 25$, find the value of $\frac{dz}{dx}$ at $x = y = a$.
18. If $u = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
19. If $u = \frac{x}{y}$, find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.
20. Write the general format of an LPP with $n$ decision variables and $m$ constraints.
21. Define vertex of a convex set. Write the importance of vertices in Linear Programming Problems containing two decision variables.
22. Show that the tangent at the point (1, 1) on the rectangular hyperbola $xy = 1$, cuts equal lengths of the axes.

**(8 × 2 = 16 Marks)**

## SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Obtain the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.
24. Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = a$.
25. If $z = \log(u^2 + v)$, $u = e^{(x^2 + y^2)}$, $v = x^2 + y$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
26. Verify Young's theorem for the function, $u = \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$.
27. If $u = e^{xyz}$, show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2)e^{xyz}$.

28. Use graphical; method to solve the LPP: Minimize $z = 3x + 10y$ subject to the following constraints: $3x + 2y \geq 6$, $4x + y \geq 4$, $2x + 3y \geq 6$, $x, y \geq 0$.

29. Use graphical method to show that the following LPP has an unbounded solution: Maximize $z = 10x + 11y$ subject to the following constraints: $x + y \geq 4, 0 \leq y \leq 3, x \geq 2, x \geq 0$.

30. If a consumer's daily income rises from Rs. 300 to Rs. 350, his purchase of a good X increases from 25 units per day to 40 units. Find the income elasticity of demand for X.

31. The demand function is given by $p = 50 - 3x$. Find TR, AR, and MR.

**(6 × 4 = 24 Marks)**

## SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). If $u = \log \sqrt{x^2 + y^2 + z^2}$, prove that $(x^2 + y^2 + z^2)(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) = 1$

ii). For the Cobb – Douglas's production function $Q = AL^{\alpha} K^{\beta}$, prove that the elasticity of substitution is unity.

33. i). f $u = \log \sqrt{x^2 + y^2}$, Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial\, u}{\partial y^2} = 0$

ii). The total demand function of a monopolist is $p = 100 - 2X$. Assuming that the demand function of segmented markets are $p_1 = 80 - 2.5x$, $p_2 = 180 - 10y$. If the cost function is $C = 50 + 40X$, maximize the profit.

34. i). If $u = x^y$, show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$

ii). An apartment complex has 250 apartments to rent. If they rent x apartments, then their monthly profit in dollar is given by $p = -8x^2 + 320x - 80{,}000$. How many apartments should they rent in order to maximize their profit.

35. A tailor has 80 sq. m. of cotton material and 120 sq. m. of woolen material. A suit requires 1 sq. m. of cotton and 3 sq. m. woolen material and a dress requires 2 sq. m. of each. A suit sells for Rs. 500 and a dress for Rs. 400. Write an LPP in terms of maximizing the income and solve it graphically.

**(2 × 15 = 30 Marks)**

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