MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :.................... **Name :.....................**

Second Semester B.Sc. Degree Examination, June 2016

First Degree Programme under CBCSS

Complementary Course: Statistics – II (for Mathematics)

AUST231.2c: Random Variables

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Define discrete random variable.
2. Find P(X = 0) for a discrete random variable with possible values – 1, 0, +1 and with $P\left(X^2 = 1\right) = \frac{2}{3}$
3. Define the distribution function F(x) of a random variable X.
4. Define the marginal distributions of the continuous random variables X and Y with joint probability density function $f(x, y)$.
5. Express the variance of a random variable X in terms of expectation.
6. State multiplication theorem on expectation for two random variables X and Y.
7. Define conditional expectation.
8. What is a scatter diagram ?
9. Correlation between two random variables X and Y= $\frac{1}{4}$, Cov (X,Y) = $\frac{1}{36}$. Obtain the product of the variances of X and Y.
10. Define fitting of a curve.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Verify whether $f(x)=\frac{x^2}{25},\ for\ \ x=0,1,2,3,4.$, is the probability mass function of a discrete random variable X.

12. Find k for a continuous random variable with p.d.f. $f(x)=k(1-x^2),\ 0<x<1$

13. Obtain the probability distribution of the random variable X with the following distribution function.

$$F(x)=\begin{cases}0, & when \quad x<1\\ \frac{3}{8}, & when \quad 1\le x<2\\ \frac{5}{8}, & when \quad 2\le x<3\\ 1, & when \quad x\ge 3\end{cases}$$

14. List any four properties of the distribution function F(x) of a random variable X.

15. If X and Y are random variables such that $Y\le X$, prove that $E(Y)\le E(X)$.

16. For two independent random variables X and Y and for real numbers a and b , prove that $V(aX-bY)=a^2V(X)+b^2V(Y)$.

17. For two random variables X and Y, prove that Cov (X + a,Y + b) = Cov(X,Y), where a and b are any two real numbers.

18. If $f(x,y)=1$; $0<x<1,\ 0<y<1$ is a joint p.d.f. Find P(x > 0.4, Y > 0.6).

19. Verify whether the random variables with the following joint p.d.f. are independent.

$$f(x,y)=2-x-y;\quad 0\le x\le 1,\ 0\le y\le 1$$
$$=0,\ otherwise.$$

20. Write a note on the "principle of least squares".

21. For two random variables X and Y, show that the geometric mean of regression coefficients is the absolute values of the coefficient of correlation.

22. For the variables X and Y, given the regression lines are $9x-4y+15=0$, and $25x-6y-7=0$. Identify the regression lines *x* on y and *y* on *x*.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Obtain the value of p for a random variable X with $E(X+2)^2 = 5$, having p.m.f.

$$f(x) = p^x(1-p)^{1-x}, x = 0,1 ,$$
$$= 0, elsewhere$$

24. A random variable X has the following probability function:

x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$ +k

Find (i). k (ii). The distribution function F(x) (iii). $P[1 < X \le 5]$

25. Define m.g.f., $M_X(t)$ of the random variable X. Prove that $\frac{d^r[M_X(t)]_{/t=0}}{dt^r}$ is the r^{th} raw moment of X.

26. Define characteristic function $\phi_X(t)$ of the random variable X. Prove that, $|\phi_X(t)| \le 1$.

27. Given the joint p.m.f. of X and Y, $f(x,y) = k(2x+y)\ x = 0,1,2;\ y = 0,1,2,3$.

Find (i). k (ii). E(Y/X = 2) .

28. If X is a random variable with p.d.f. $f(x) = 1,\ 0 \le x \le 1;\quad f(x) = 0,\ otherwise$. Find the probability density function of $Y = -2\log_e X$.

29. Explain the method of fitting of a curve of the form $y = ab^X$.

30. State and prove Cauchy – Schwartz Inequality.

31. Prove that the coefficient of correlation remains unchanged under linear transformation.

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). Define raw moments and central moments of a random variable X. Express r^{th} central moment in terms of raw moments. Deduce the expression for first 4 central moments in terms of raw moment **(9 Marks)**

ii). Prove that two random variables X and Y are independent if and only if $M_{X,Y}(t_1,t_2) = M_X(t_1)M_Y(t_2)$ **(6 Marks)**

33. Let X and Y are two random variables with joint p.m.f. $f(x,y) = \frac{x+2y}{18}$ $x = 1,2;\ y = 1,2$

i). Find Correlation between X and Y **(9 Marks)**

ii). Find V(X/Y=2). **(6 Marks)**

34. i). Derive the regression lines *x on y* and *y on x* using a set of observations $(x_1, y_1),...,(x_n, y_n)$ on the variables X and Y. **(10 Marks)**

ii). Derive an expression for the angle θ between the regression lines. **(5 Marks)**

35. i). Derive Spearman's coefficient of correlation between two variables X and Y. **(8 Marks)**

ii). Find the rank correlation coefficient for the following data:

X:	45	43	46	42	40	44	49
Y:	42	39	47	40	41	43	45

(7 Marks)

(2 × 15 = 30 Marks)

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