



**MAR IVANIOS COLLEGE (AUTONOMOUS)**  
**THIRUVANANTHAPURAM**

Reg. No. : .....

Name : .....

**First Semester B.A. Degree Examination, November 2015**

**First Degree Programme under CBCSS**

**Complementary Course: Mathematics – I (for Economics)**

**AUMM131.1a: Mathematics for Economics – I**

(for 2015 Admissions Only)

Time: 3 Hours

Max. Marks: 80

**SECTION – A**

*Answer ALL questions / problems in one or two sentences.*

1. Write an explicit function from the relation  $x^2 + y^2 = 16$ .
2. Find  $\lim_{x \rightarrow 5} \left[ \frac{x^3 - 125}{x - 5} \right]$ .
3. Find the derivative of  $y = x^3 (5x^2 - 1)$ .
4. What is the general form of an implicit linear function in  $x$  and  $y$ .
5. State the quotient rule of differentiation.
6. Find the second derivative  $\sqrt{x}$ .
7. Find a relation between the variables  $x$  and  $y$  when  $x = 3t^2$ ,  $y = 6t$ .
8. What is the geometrical meaning of derivative of a function of a point ?
9. What is the derivative of  $\log x$ .
10. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{e^{5x} - 1}{x} \right)$ .

(10 × 1 = 10 Marks)

**SECTION – B**

*Answer any EIGHT questions / problems, not exceeding a paragraph.*

11. Find the natural domain of the function  $f(x) = 2 + \sqrt{x-1}$ .

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12. Draw the graph of the function  $y = \frac{3}{x^2}$ .
13. Evaluate  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$ .
14. A fixed plant is used to manufacture radio sets and, if  $x$  sets are turned out per week, the total variable cost is Rs.  $\left(3x + \frac{1}{25}x^2\right)$ . Show that the average variable cost increases steadily with output.
15. Find  $\frac{dy}{dx}$  if
- $x^2 + y^2 = 1$
  - $x = t^3 - 1, y = t^2 + 1$
16. Differentiate with respect to  $x$
- $y = \frac{1+x}{1-2x}$
  - $y = \frac{1}{\sqrt{1-x^2}}$
17. Discuss the continuity of the function  $f(x) = \frac{1-x}{1+x}$ .
18. For what values of  $x$  there is a gap in the graph of  $y = \frac{x^2 - 9}{x^2 - 5x + 6}$ .
19. If the demand law is  $p = \frac{a}{x} - c$ , show that the total revenue decreases as output increases, marginal revenue being a negative constant.
20. Find the slope of the tangent to the curve  $y = 5x^2 - 7x + 9$  at the point  $x = 2$ .
21. Find  $\frac{dy}{dx}$  if
- $y = (2x^2 + 1) \ln(x + 1)$
  - $y = \ln 5x^3$
22. If  $x$  and  $y$  satisfies the relation  $xy = a$ , show that  $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0$ .

(8 × 2 = 16 Marks)

### SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. What are the conditions necessary for the continuity of a function ? Find the value of  $k$  so that  $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$  is a continuous function.
24. A firm produces an output of  $x$  tons of a certain product at a total variable cost given by  $\Pi = ax^3 - bx^2 + cx$ . Show that the average cost curve is parabola. Also find the output for least average cost and the corresponding value of average cost.
25. Plot the graph of  $y = \frac{2x}{x^2 + 1}$  for positive values of  $x$ , and show how the graph for negative values of  $x$  can be deduced. What are the greatest and least values of  $y$  ?
26. If  $y = 3x^4 - 2x^3 + x^2 - 4x + 2$  find  $\frac{d^2 y}{dx^2}$ .
27. Show that  $(x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1}) = 1$  and deduce that  $\log(x + \sqrt{x^2 - 1}) = -\log(x - \sqrt{x^2 - 1})$ .
28. Find the function inverse to  $y = \frac{2x + 1}{x - 1}$  and show that it is single valued. Represent it graphically and give some account of the behaviour of the graph in the neighborhood of  $x = 1$  and of  $y = 2$ .
29. By algebraic methods, solve  $2x - y + 3 = 0$  and  $xy = 2$ .
30. Discuss briefly the various cases of the limits of  $f(x)$  as  $x \rightarrow \pm\infty$ .
31. If  $f(x) = 4 + \frac{1}{1 + \frac{1}{(1-x)}}$ ,  $0 < x < 1$ , find  $\lim_{x \rightarrow 1} f(x)$ .

(6 × 4 = 24 Marks)

### SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). If  $f(x) = x^3$ ,  $g(x) = \sin x$ , find  $f \circ g(x)$  and  $g \circ f(x)$ .
- ii). Find the domain and range of the function  $y = \frac{x + 1}{x - 1}$ .

iii). If  $f(x) = x^2 + 1$ , find  $f(x + 2)$  and  $f(\frac{1}{x})$ .

33. i). Find the point on the curve  $y = x^2 - 5x + 6$  where the tangent is parallel to the x axis.

ii). If  $y = x^2 + \frac{1}{x^2}$ , show that  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 4y = 0$ .

34. i). Explain the concepts of the total revenue curve and average revenue curves.

ii). A gramophone sells at Rs.  $p$  and it is known that the demand,  $x$  hundred machines per year, is given by  $x = \frac{90}{p+5} - 6$ . Plot the graph of the demand curve. At what price does the demand tend to vanish? Draw the graph of the total revenue curve and determine at what price and output the total revenue is greatest.

35. i). Find, from the definition, the derivative of  $x^3$ .

ii). Find  $\frac{dy}{dx}$  if

(a).  $y = (3x^2 + 6)(2x - \frac{1}{4})$

(b).  $y = \sqrt{\frac{1+x}{1-x}}$

**(2 × 15 = 30 Marks)**

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