MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :…… **Name :…….**

Second Semester B.Sc. Degree Examination, June 2016
First Degree Programme under CBCSS
Complementary Course: Mathematics – II (for Chemistry and Physics)
AUMM231.2b / AUMM231.2d: Integration, Power Series and Linear Algebra

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Find the average value of $f(x) = \cos x$ over $\left[0, \frac{\pi}{2}\right]$.
2. Suppose that a particle moves so that its velocity at time t is $V(t) = \sin 2t + 2$ m/s. Find the displacement of the particle during the time interval $0 \leq t \leq \pi$.
3. Evaluate $\int_0^a \int_0^b x(x+y)dydx$.
4. Evaluate $\int_0^1 \int_0^2 e^{x+y}dydx$.
5. Suppose that g(x) is a function for which $\int_1^2 g(x)\,dx = -2$. Then find $\int_1^2 5g(x)dx$.
6. Find the Maclaurin polynomials p_0, p_1 and p_2 for the function $f(x) = e^x$.
7. Find the first four Taylor polynomials for $ln\ x$ about $x = 2$.
8. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 4 & 3 \end{bmatrix}$.
9. Write a matrix with eigen values 1, 2, 3.
10. Give an example of a matrix which is in the row echelon form.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems. Each question carries 2 marks.*

11. Find the area under the graph of $y = x^2$ over the interval [0,3].
12. The area under the graph of $y = f(x)$ is $A(x) = x + e^x - 1$. Find the function f.
13. Solve the initial value problem $\frac{dy}{dx} = x^2\sqrt{x^3}, y(0) = 0$.
14. Find $\int sin^3 2\theta \, d\theta$.
15. Evaluate $\int_{-1}^{1} |x| dx$.
16. A penny is released from rest near the top of the Empire State Building at a point that is 1250 ft above the ground (draw a suitable figure). Assuming that the free – fall model applies, how long does it take for the penny to hit the ground, and what is its speed at the time of impact ?
17. Evaluate $\int_0^{ln2} \int_0^1 xye^{y^2x} dydx$.
18. Find the Taylor series for $\frac{1}{x}$ about $x = 1$.
19. Approximate $sin\, 85^0$ to four decimal place accuracy using an appropriate Taylor series.
20. Reduce the matrix $\begin{bmatrix} -1 & 4 & 6 \\ 2 & 3 & -5 \\ 7 & 1 & 1 \end{bmatrix}$ into row echelon form.
21. Show that a matrix and its transpose have the same eigen values.
22. Solve $4x_1 + x_2 - 3x_3 + x_4 = 0$
$2x_1 - x_3 = 0$

(8 × 2 = 16 Marks)

SECTION – C

*Answer any **SIX** questions. Each question carries 4 marks.*

23. Find the area enclosed by the curve $x^{2/3} + y^{2/3} = a^{2/3}$.
24. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x – axis to generate a solid. Find its volume.
25. Evaluate the integral by reversing the order of integration $\int_0^2 \int_{y/2}^1 \cos(x^2)\ dxdy$.
26. Use a polar double integral to find the area enclosed by the three – petaled rose $r = sin\, 3\theta$.
27. Find the sum of the series $\sum_{k=1}^{\infty}(\frac{3}{4^k} - \frac{2}{5^{k-1}})$.

28. Find the radius of convergence and the interval of convergence of the power series.

$$\sum_{k=1}^{\infty} \frac{x^k}{k(k+1)}$$

29. Find the rank of the matrix by reducing it into the Echelon form.

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & -1 \\ 3 & 0 & 0 \end{bmatrix}$$

30. Find the general solution of the system $x_1 - 3x_2 + x_3 - 7x_4 + 4x_5 = 0$

$$x_1 + 2x_2 - 3x_3 = 0$$

$$x_2 - 4x_3 + x_5 = 0$$

31. Find the eigen values and the corresponding eigen vectors of the matrix.

$$\begin{bmatrix} 2 & 0 & -1 \\ -6 & 7 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

(6 × 4 = 24 Marks)

SECTION – D

*Answer any **TWO** questions. Each question carries 15 marks.*

32. i). Sketch the region enclosed by the curves $y = x^2$ and $y = x + 2$ and find its area.

ii). Use cylindrical shells to find the volume of the solid generated when the region bounded by the curve $y = x^3$, $x = 1$ and $y = 0$ is revolved about y – axis.

33. i). Evaluate the integral by converting to polar coordinates

$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dxdy.$

ii). Use a triple integral to find the volume of the solid in the first octant bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$.

34. i). Approximate the integral $\int_0^1 e^{-x^2} dx$ to three – decimal place accuracy using Taylor's series.

ii). Find the first three non zero terms of the Maclaurin series for $\tan x$.

35. i). Diagonalize the matrix $\begin{bmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, if possible.

ii). Prove that A is diagonalizable if A^2 is diagonalizable.

(2 × 15 = 30 Marks)

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