MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :…… **Name :…….**

Second Semester B.A. Degree Examination, June 2015

First Degree Programme under CBCSS

Complementary Course: Mathematics – II (for Economics)

AUMM231.1a: Mathematics for Economics – II

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Find the second derivative of $y = (5x + 9)^3$.
2. Determine whether the function $y = 4x - x^2$ is increasing or decreasing at $x = 1$.
3. If $y = \sqrt{5 - x^2}$, find $\frac{dy}{dx}$.
4. What is the marginal revenue function for the demand $p = a - bx$.
5. The total revenue for a firm is $R = 22q - q^2$. What is the output at which the total revenue is maximum ?
6. Write down the degree of the homogeneous function $z = \frac{x^3 + y^3}{x - y}$.
7. Find the turning point of the function $f(x) = x^2 - 4x + 8$.
8. Write the function $x^2 - xz + yz - z = 0$ in explicit form.
9. Define partial derivative of $z = f(x,y)$ with respect to y at the point (x,y).
10. Find the second order partial derivative with respect to y of the function $x^3 + y^3 - 3xy$.

(10 x 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Find the minimum value of $y = 2x - 1 + \frac{1}{x}$ for positive values of x.

12. Show that $y = \frac{1}{2x+1}$ is a monotonic function. Illustrate by drawing the graph of the function.

13. If $xy = 5 - 2x$, show that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$.

14. If $f(x) = ax + b$, find the derivatives of $x\,f(x)$ and $\frac{f(x)}{x}$.

15. Find $\frac{dy}{dx}$ when $x = ct$, $y = \frac{c}{t}$.

16. Explain about inflexional value of the function $f(x)$ at $x = a$.

17. Show that the demand curve $p = \frac{a}{x+b} - c$ is downward sloping and convex from below.

18. If $z = (x^2 + y^2)^2$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

19. State Young's theorem in partial derivatives.

20. Find the first and second order partial derivatives with respect to x of $\log\left(\frac{x}{x+y}\right)$.

21. If $x^2 + y^2 - 2x + 4y + 1 = 0$, find $\frac{\partial z}{\partial x}$.

22. State the complete differential form of the function $z = f(x, y)$.

(8 x 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Obtain the extreme values of $f(x) = x^3 - 3x^2 + 5$.

24. Evaluate the differential of $z = x^3 + y^3 - 3xy$.

25. Show that the perimeter of a rectangle of area 16 square meters is least when the rectangle is a square of side 4 meters.

26. Find $\frac{dz}{dt}$, when $z = x^2 + y^2$, where $x = 1 + t$, $y = 1 - t$.

27. If $x^3 + y^3 + z^3 - 3xy = 0$ defines z as a function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

28. Find x as a function p from the demand law $p = \sqrt{a - bx}$. Show that the demand curve is an arc of a parabola with its axis parallel to the x – axis.

29. Show that $z = 2x + y + x^2 + xy - y^2$ has a single stationary value. Verify that it is a maximum value.

30. Examine the sections of the surface $x^2 + y^2 + z^2 = a^2$ and show that it is a sphere of radius a.

31. For inputs x and y, let P = xy be the production function. The budget constraint is $2x + y = 6$. How much x and y should be bought to obtain maximum output P.

(6 x 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). If $u = \log(x^2 + y^2)$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

    ii). For the total cost function $\Pi = ax^2 + bx + c$, explain about average cost and marginal cost. Give a graphical illustration.

33. i). If $y = x^2 + \frac{1}{x^2}$, show that $x^2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} - 4y = 0$.

    ii). A television company produces x sets per week at a total cost of Rs. $\left(\frac{1}{25}x^2 + 3x + 100\right)$. The company is a monopolist and the demand is $x = 75 - 3p$, where p is the price per set. What is the monopoly price? Illustrate this by drawing a graph.

34. i). Show that $y = x + \frac{1}{x}$ has a maximum and a minimum value. Draw a graph to illustrate.

ii). The average cost function for producing and marketing x units of a product is AC = $2x - 11 + \frac{50}{x}$. Find the total cost function and the marginal cost function.

35. i). Show that the curve $y = \frac{2x}{x^2 + 1}$ has three points of inflexion separated by a maximum point and a minimum point.

ii). State Euler's theorem on homogenous functions. Verify the theorem for $z = x^2 - xy + 2y^2$.

(2 x 15 = 30 Marks)

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