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MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :………………….**

Fourth Semester B.Sc. Degree Examination, June 2016

First Degree Programme under CBCSS

Core Course: Mathematics – III

AUMM441: Algebra and Calculus II

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences. Each question carries 1 mark.*

1. Find $(x + 2)^3$ in $Z_2[x]$.
2. If $f(x) = 2x^3 + 4x^2 + 3x + 2$ and $g(x) = 3x^4 + 2x + 4$ find $f(x).g(x)$ in $Z_5(x)$.
3. What are the units of the ring $Z_7[x]$.
4. Find the remainder in $Q[x]$ when $x^{40} - 8x^{12} + 3$ is dived by $x^4 - 1$
5. Find a zero of $x^4 - 2x^2 - 2$ in $Q[x]$.
6. Find $\lim_{(x,y)\to(4,-2)}(4xy^2 - x)$.
7. Show that the value of $\frac{xyz}{x^4+y^4+z^4}$ approaches zero as $(x, y, z) \to (0, 0, 0)$ along $x = at$, $y = bt$, $z = ct$.
8. If $f(x, y) = x^4y^2 - \sin(xy) + 2x^7$ find f_{xyy}.
9. Find the slope of the surface $Z = f(x,y)$ in the x–direction at the point (3,0).
10. Evaluate $\iint_1^2 x\cos(xy)\,dy\,dx$. where x ranges from $\frac{\pi}{2}$ to π

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems. Each question carries 2 marks.*

11. In $Q[x]$, when $f(x)$ is divided by $(x^2 - 3)(x + 1)$, the remainder is $x^2 + 2x + 5$. What is the remainder when $f(x)$ is divided by $x^2 - 3$?
12. Factorize $4x^2 - 4x + 8$ in $Z_3[x]$.

13. If R is a commutative Ring with unit element prove that so is $R[x]$.
14. Define an irreducible polynomial over a field F. Show that $x^3 + x + 1$ is irreducible over Z_2.
15. Using Euclid' s algorithm find a g.c.d of $x^2 - x+4$ and $x^3 + 2x^2 + 3x + 2$ in $F_3[x]$.
16. At what rate is the volume of the box changing if its length is 8 ft. and is increasing at 3ft/s, its width is 6 ft. and is increasing at 2 ft/s. and its height is 4ft. and is increasing at 1ft/s.
17. Locate all the relative extrema and saddle points of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$.
18. Find a point on the surface $Z = 8 - 3x^2 - 2y^2$ at which the tangent plane is perpendicular to the line x = 2 – 3t, y = 7 + 8t, z = 5 – t.
19. Find $\frac{dw}{dt}$ using chain rule, $w = 5x^2 y^3 z^4$, $x = t^2$, $y = t^3$, $z = t^5$.
20. Show that the function $U = \ln(x^2+y^2)$, $V = 2\tan^{-1}(\frac{y}{x})$ satisfies the Caushy Reimann equations $U_x = V_y$, $U_y = -V_x$.
21. Use double integral to find the volume of the solid bounded by the plane $z = 4 - x - y$ and below by the rectangle $R = [0,1] \times [0,2]$.
22. Evaluate $\iint_R (2x - y^2)dA$ over the triangular region R enclosed between the lines $y = -x + 1$, $y = x + 1$ and $y = 3$.

(8 × 2 = 16 Marks)

SECTION – C

*Answer any **SIX** questions. Each question carries 4 marks.*

23. Find a solution of $x^4 = 20x + 150$ by Ferrari's method.
24. Find a solution of $y^3 + 3y = 5$ by cardon's method.
25. State and prove Euler's real version of the fundamental theorem of algebra.
26. For any n, prove that $\sum_{d/n} \phi(d) = n$
27. If p is irreducible and f is any polynomial which is not divisible by p, show that the greatest common divisor of p and f is 1.
28. Show that $f(x, y) = 3x^2y^5$ and $g(x, y) = \sin(3x^2 y^5)$ are continous everywhere.
29. If $f(x, y) = x \sin(xy^3)$ find $f_x(x,y)$ and $f_y(x,y)$
30. Use double integral to find the area of the region R enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$.

31. Evaluate $\iint_{y}^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dx\, dy$ where y ranges from 0 to $\sqrt{2}$.

(6 × 4 = 24 Marks)

SECTION – D

*Answer any **TWO** questions. Each question carries 15 marks.*

32. i). State and prove the division algorithm in $F[x]$ where F is a field.

    ii). Prove that $a \in F$ is a zero of $f(x) \in F[x]$ if and only if $x - a$ is a factor of $f(x)$ in $F[x]$.

33. i). State and prove the unique factorization theorem in $F[x]$.

    ii). Factorize $(x^4 + 3x^3 + 2x + 4) \in Z_5[x]$.

34. i). Find the absolute maximum and minimum values of $F(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0,0), (3,0) and (0,5).

    ii). Use Langrange Multiplier Method to find the points on the circle $x^2 + y^2 = 45$ that are closest to and farthest from (1, 2).

35. i). Find the volume of the prison whose base is the triangle in the xy plane bounded by the x – axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$

    ii). Find the Surface area of that portion of the surface $z = \sqrt{4 - x^2}$ that lies above the rectangle R in the xy plane whose coordinates satisfy $0 \leq x \leq 1$ and $0 \leq y \leq 4$.

(2 × 15 = 30 Marks)

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