



**MAR IVANIOS COLLEGE (AUTONOMOUS)**  
**THIRUVANANTHAPURAM**

Reg. No. : .....

Name : .....

**Second Semester B.Sc. Degree Examination, June 2015**

**First Degree Programme under CBCSS**

**Foundation Course – II: (for Mathematics)**

**AUMM221: Foundations of Mathematics**

Time: 3 Hours

Max. Marks: 80

**SECTION – A**

*Answer ALL questions / problems in one or two sentences.*

1. Find a zero divisor of  $\mathbb{Z} / 4\mathbb{Z}$ .
2. If  $a \equiv b \pmod{1001}$ , then check whether  $a \equiv b \pmod{7}$ .
3. Compute  $13 \cdot 19$  modulo 23.
4. Determine the open interval on which  $f(x) = x^2$  is concave.
5. Find the point of inflection (if any) of  $f(x) = (x - a)^3$ , where  $a$  is a constant.
6. Find the critical values of  $f(x) = 2x^3 - 15x^2 + 36x$ .
7. Evaluate  $\int \tan x \, dx$ .
8. Find the average value of  $f(x) = x^2 + 1$  over  $[0, 2]$ .
9. Find the rectangular coordinates of the point P whose polar coordinates are  $(6, \pi/3)$ .
10. Find the eccentricity and the distance from the pole to the directrix of the conic

$$r = \frac{3}{2 - 2 \cos \theta}.$$

**(10 x 1 = 10 Marks)**

**SECTION – B**

*Answer any EIGHT questions / problems, not exceeding a paragraph.*

11. Define a *unit element* of a commutative ring. Find all the unit elements of  $\mathbb{Z}$ .
12. Find the inverse of  $[3]_{17}$  in  $\mathbb{Z} / 17\mathbb{Z}$ .

P.T.O.

13. In  $\mathbb{Z}/13\mathbb{Z}$ , find one solution of the equation,  $[4]X = [7]$ .
14. Find the order of 2 modulo 7.
15. Write down the addition and multiplication table for arithmetic modulo 5.
16. If  $p$  is prime then prove that  $\phi(p) = p - 1$ .
17. Find all absolute extrema of the function  $f(x) = x^3 - 3x^2 + 4$  on the interval  $(-\infty, \infty)$ .
18. Evaluate  $\int \sin^2 x \cos x \, dx$ .
19. Suppose that a particle moves on a coordinate line so that its velocity at time  $t$  is  $v(t) = (t^2 - 2t) \text{ m/s}$ , then find the displacement of the particle during the time interval  $0 \leq t \leq 3$ .
20. Evaluate  $\int_0^{\pi/2} \sqrt{1 + \sin 2x} \, dx$ .
21. Use cylindrical shell to find the volume of the solid generated when the region  $R$  in the first quadrant enclosed between  $y = x$  and  $y = x^2$ .
22. Find the entire area within the cardioid  $r = 1 - \cos \theta$ .

(8 x 2 = 16 Marks)

### SECTION – C

*Short essay type problems : Answer any SIX questions.*

23. Show that  $2^{340} \equiv 1 \pmod{341}$ .
24. Suppose  $f(x) = x^4 + 5x^3 + 8x^2 + x + 15$ , compute  $f(12)$  modulo 17.
25. If  $X = [5]_{16}$  is a solution of  $[6]_{16}X = [14]_{16}$ , find all other solutions.
26. Find the average value of the function  $f(x) = \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}$  over  $\left[\frac{\pi}{2}, \pi\right]$ .
27. Find  $\frac{d}{dx}[\ln |x|]$ .
28. Find the area of the region enclosed by  $x = y^2$  and  $y = x - 2$ .
29. Show that  $\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$ .
30. Evaluate  $\int \frac{2x+4}{x^3-2x^2} \, dx$ .
31. Evaluate the improper integral  $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ .

(6 x 4 = 24 Marks)

## SECTION – D

*Long essay type problems : Answer any TWO questions.*

32. i). State and prove Fermat's theorem.  
 ii). If  $e$  is the order of  $a$  modulo  $m$ , and  $a^f \equiv 1 \pmod{m}$ , prove that  $e$  divides  $f$ .
33. i). Prove that  $\mathbb{Z}/m\mathbb{Z}$  is a field if and only if  $m$  is prime.  
 ii). If  $p$  is a prime and  $a$  is not divisible by  $p$ , then prove that the order of  $a$  modulo  $p$  divides  $p - 1$ .
34. i). Evaluate  $\int_0^1 \tan^{-1}(x) dx$ .  
 ii). Find the length of the curve  $24xy = y^4 + 48$  from  $y = 2$  to  $y = 4$ .
35. i). State the horizontal line test. Check whether the functions  
 a)  $f(x) = x^2$   
 b)  $f(x) = x$  and  
 c)  $f(x) = 2x + 5$  have an inverse by sketching the graph.
- ii). Sketch the graph  $r = \frac{2}{1 - \cos \theta}$  in polar coordinates.

**(2 x 15 = 30 Marks)**

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