MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :………………….**

Third Semester B.Sc. Degree Examination, November 2016
First Degree Programme under CBCSS
Complementary Course: Mathematics – III (for Chemistry and Physics)
AUMM331.2b/ AUMM331.2d: Vectors and Differential Equations

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Find $\vec{r}^{\,I}(0)$ if $\vec{r}(t) = (3\sin t)\,\vec{i} - (2t)\,\vec{j}$
2. Find the unit tangent vector to the curve $\vec{r}(t) = t^2\,\vec{i} + t^3\,\vec{j}$ at the point where t = 2
3. Define gradient of a function f(x,y,z).
4. If $F(x,y,z) = x^2\,\vec{i} - 2\vec{j} + yz\vec{k}$ find *curl*(F).
5. State " Divergence theorem"
6. Prove that $F(x,y,z) = e^y\,\vec{i} + xe^y\,\vec{j}$ represents a conservative vector field on the XY-plane
7. Evaluate the line integral $\int_c (x^2 - y)\,dx + x\,dy$ where c is the circle $x^2 + y^2 = 4$
8. Solve the differential equation $3\frac{dy}{dx} = \frac{4x}{y^2}$
9. Write the characteristic equation of the ordinary differential equation ,

   $3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 2x^2$
10. Write the general form of a first order differential equation and its general solution

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Find the angle between the tangent lines of the curves $\vec{r}(t)$ = $t^2\,\vec{\imath}$ + $t\,\vec{\jmath}$ + $3t^3\,\vec{k}$ and $\vec{s}(t)$ = (t-1) $\vec{\imath}$ + $\frac{t^2}{4}\vec{\jmath}$ +(5-t) $\vec{k}$ the point (1, 1, 3)
12. Verify whether $\vec{r}(t) = te^{-t}\vec{\imath}$ + (t^2 -2t) $\vec{\jmath} + cos\pi t\,\vec{k}$ is a smooth curve
13. Find the position and velocity of a particle whose acceleration is given by $\vec{a}(t)$ = sint $\vec{\imath}$ + cost $\vec{\jmath}$ + $e^t\,\vec{k}$ with $\vec{v}(0) = \vec{k}$ and $\vec{r}(0)$ = -$\vec{\imath} + \vec{k}$
14. Find the gradient of f(x,y) =$(x^2 + xy)^3$ at the point (-1, -1)
15. If $\vec{r}(t)$ = x $\vec{\imath}$ + y $\vec{\jmath} + z\,\vec{k}$, prove that ∇(IIrII) = $\frac{r}{IIrII}$.
16. Evaluate the line integral $\int_c (3x + 2y)\,dx$ + (2x – y) dy along the line segment from (0, 0) to (1, 1).
17. State Green's theorem for a simply connected plane region.
18. Find the work performed by a vector field $\vec{F}$ on a particle around a smooth curve C using Stokes theorem.
19. Find the general solution of the differential equation $y^{I} = \frac{y}{x+y}$
20. Solve; $xy^{I} = \frac{y^2}{x}$ + y
21. Find two independent solution of the differential equation $\frac{d^2y}{dx^2}$ + y=0
22. Solve: $y^{II} - y^{I} - 6y = 0$

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Find the radius of curvature of $\vec{r}(t) = e^t$cost $\vec{\imath}$ + e^tsint $\vec{\jmath}$ + $e^t\,\vec{k}$ at t = 0
24. Find the directional derivative of f(x, y, z) = $\frac{y}{x+y}$ at P(2,1,-1) in the direction of $\overrightarrow{PQ}$ where Q = (-1,2,0)
25. Find the work done by the force field $\vec{F}(x, y)$ = xy $\vec{\imath}$ + $yz\,\vec{\jmath}$ + $xz\,\vec{k}$ along the curve $\vec{r}(t)$ =t $\vec{\imath}$ + $t^2\vec{\jmath}$ + $t^3\vec{k}$ when $0 \le t \le 1$.

26. Evaluate $\int_c 3xy\,dx + 2xy\,dy$ using Green's theorem where C is the rectangle bounded by x = - 2 , x = 4, y = 1 and y = 2.
27. Evaluate the surface integral $\iint_\sigma f(x,y,z)\,dS$ where f(x, y, z) = x – y - z and σ is the portion of the plane x + y = 1 in the first octant between z = 0 and z = 1.
28. Find the flux of $\vec{F}(xy) = x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k}$ across the surface of the cylindrical solid bounded by $x^2 + y^2 = 4$, z = 0, & z = 3 using divergence theorem.
29. Solve the intial value problem $y^{ll} - 2y^{l} + 2y = 0$ with y (0) = 6 & $y^{l}(0) = 1$.
30. Find the particular integral of $y^{ll} + 2y^{l} - 3y = 4e^{2x}$.
31. Solve: $y^{l} + xy = xy^2$

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. a) Find the arc length of the curve $\vec{r}$ (t) = (3cost)$\vec{\imath}$ + (3sint) $\vec{\jmath}$ + t $\vec{k}$, 0 ≤ t ≤ 2π.
    b) Find the distance travelled and displacement of a particle from time t = 1 to t =5 whose position vector at 't' is given by $\vec{r}$ (t) = (4cos π t) $\vec{\imath}$ + (4sin π t) $\vec{\jmath}$ + t $\vec{k}$.
33. a) Find) ∇. (F × G) if F(x,y,z) = 2x $\vec{\imath}$ + $\vec{\jmath}$ + 4y $\vec{k}$ and G (x,y,z) = x $\vec{\imath}$ + y $\vec{\jmath}$ – z $\vec{k}$.
    b) Show that $\vec{F}$ (x,y) = $(2xy^3)\vec{\imath} + (1 + 3x^2y^2)\vec{\jmath}$ is a conservative vector field on the xy plane. Find the potential function corresponding to $\vec{F}$.
34. Verify Stoke's theorem for $\vec{F}$ (x,y,z) = x $\vec{\imath}$ + y $\vec{\jmath}$ + z $\vec{k}$ where σ is the upper hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ (upward orientation).
35. a) Find the orthogonal trajectories of the family of curves $x^2 + 2y^2 = k$.
    b) Find the general solution of $y^{ll} + 4y = 8x$.

(2 × 15 = 30 Marks)

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