MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :…… **Name :…….**

Second Semester B.Sc. Degree Examination, June 2015

First Degree Programme under CBCSS

Complementary Course: Statistics – II (for Mathematics)

AUST231.2c: Random Variables

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Define distribution function and mention one of its properties ?
2. Define raw and central moments ?
3. An experiment consists of 3 independent tosses of a fair coin and let X denote the number of heads occurred, find the probability mass function of X ?
4. Define independence of random variables ?
5. State the multiplication theorem on expectation ?
6. Describe characteristics function and specify any one of its properties ?
7. Describe the principle of least square ?
8. Define correlation coefficient ?
9. Describe the regression lines ?
10. State Cauchy – Schwartz inequality ?

(10 x 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. A random variable X has the pdf $f(x)=\frac{1}{4}, -2<x<2$ and zero otherwise.

    Find i). $P(X<1)$ ii). $P(|X|<1)$?

12. Describe marginal and conditional distributions ?

13. Let X be a random variable with pdf $f(x) = 2x, 0 < x < 1$ and zero otherwise. Find the pdf of Y = 3X + 1 ?
14. Establish with the help of an example that the expectation of a random variable need not always exist ?
15. Prove that the moment generating function of the sum of two independent random variables is the product of their individual moment generating functions ?
16. Define variance and prove that $E(X - c)^2 = V(X) + (E(X) - c)^2$?
17. Explain the method for fitting a curve of the form $y = ae^{bx}$?
18. Prove that the covariance is independent of change of origin ?
19. Explain the use of scatter diagram ?
20. Two regression coefficients are 1.6 and 0.1. Find r ?
21. Let the random variables X and Y, take values 0, 1 and 2 and have the joint pmf $f(x, y) = \frac{1}{3}; (x, y) = (0,0), (1,1), (2,2),$ zero otherwise. Find the marginal distributions of X and Y ?
22. With usual notations show that E (E(X / Y)) = E(X) ?

(8 x 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. A random variable X has the pdf

$$f(x) = \begin{cases} 0, \text{ if } x < 0 \text{ or } x > 2 \\ x, \text{ if } 0 < x \le 1 \\ \frac{1}{2}, \text{ if } 1 < x \le 2 \end{cases}$$

Find i). $P(\frac{1}{2} < x < \frac{3}{2})$ ii). $P(X > 1)$ iii). $P(X < 1)$

24. If $f(x, y) = 2, 0 < x < 1, 0 < y < x,$ find the conditional distributions ?
25. A continuous random variable X has the distribution function

$$F(x) = \begin{cases} 0 \text{ if } x \le 1 \\ k(x-1)^4 \text{ if } 1 < x \le 3 \\ 1 \text{ if } x > 3 \end{cases}$$

Find i). k ii). $f(x)$ and iii). $P(X > 2)$

26. What is the expected number of failures preceding the first success in an infinite series of Bernoulli trials with constant probability of success in each trial ?
27. Define moment generating function (mgf) of a random variable X. Find the mgf of X with probability function $f(x) = ae^{-ax}, x > 0$
28. If X and Y are two random variables, prove that
    $V(aX - bY) = a^2V(X) + b^2V(Y) - 2ab\,Cov(X, Y)$
29. Explain the method of fitting a parabola ?
30. With usual notations show that the correlation coefficient lies between – 1 and + 1?
31. Describe the equations to the lines of regression and show that the coefficient of correlation is the GM of the regression coefficients ?

(6 x 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). X and Y have a bivariate distribution given by $P(X = x, Y = y) = (x + 3y)/24$, where $(x, y) = (1, 1), (2,1), (1, 2), (2, 2)$. Find the marginal distributions and conditional distributions of X given $Y = 2$ and Y given $X = 1$. **(7 Marks)**

    ii). Find the Spearmann's rank correlation coefficient for the following data.

X:	68	64	75	50	64	80	75	40	55	64
Y:	62	58	68	45	81	60	68	48	50	70

(8 Marks)

33. i). Find the regression equations of X on Y and Y on X for the following data and hence find the correlation coefficient.

X:	1	2	3	4	5	6	7	8	9
Y:	9	8	10	12	11	13	14	16	15.

    Also find the value of Y when X = 6.2 **(10 Marks)**

    ii). Fit a curve of the form $Y = ab^x$ for the following data

X:	1	2	3	4	5	6	7	8
Y:	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

(5 Marks)

34. Let X be a random variable with pdf
    $f(x) = kx^2 e^{-x}$, $x > 0$ and zero otherwise. Find
    i). the value of k ii). Mean iii). Variance
    iv). Third and fourth central moments v). Skewness and Kurtosis

(15 Marks)

35. Two random variables X and Y have the following probability density function: $f(x,y) = 2 - x - y$; $0 \leq x \leq 1$, $0 \leq y \leq 1$ and zero otherwise. Find
    i). Marginal and conditional distributions
    ii). Mean and variance of X and Y
    iii). Covariance between X and Y and the correlation between X and Y.

(15 Marks)

(2 x 15 = 30 Marks)

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