



MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, November 2015

First Degree Programme under CBCSS

Complementary Course: Mathematics – III (for Physics)

**AUMM331.2d: Differential Equations, Theory of Equations and
Theory of Matrices**

Time: 3 Hours

Max. Marks: 80

SECTION – A

Answer ALL questions / problems in one or two sentences.

1. Define exact differential equation.
2. Solve $y' + (5 - y)x = 0$.
3. Solve $y'' - 2y' = 0$.
4. Define the rank of a matrix.
5. Define diagonalization of a matrix.
6. Find the reduced form of the matrix $\begin{bmatrix} -3 & 1 \\ 2 & 2 \\ 4 & -3 \end{bmatrix}$.
7. What is the characteristic equation of the matrix $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$.
8. State the fundamental theorem of algebra.
9. If a, b, c are the roots of $5x^3 + 6x^2 - x + 1 = 0$, then what is the value of $bc + ca + ab$?
10. Find the cubic equation whose roots are 3, 2, and -16 .

(10 × 1 = 10 Marks)

SECTION – B

Answer any EIGHT questions / problems, not exceeding a paragraph.

11. Solve $2xy + 6x + (x^2 - 4)y' = 0$.
12. Solve $(3x^2y^2 + x^2) dx + (2x^3y + y^2) dy = 0$.

13. Solve $(x^2 + 1) \frac{dy}{dx} + 4xy = x$.
14. Solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0$, $y(0) = 3$, $y'(0) = -1$.
15. Solve $6x_1 - x_2 + x_3 = 0$, $x_1 - x_4 + 2x_5 = 0$, $x_1 - 2x_5 = 0$.
16. Find the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.
17. Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 13 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by reducing it into echelon form.
18. Show that matrix A and A^T have same eigen values.
19. Solve the equation $x^3 - 4x^2 - 20x + 48 = 0$ given that two of its roots a, b are connected by the equation $a + 2b = 0$.
20. Show that if the roots of the equation $x^3 - lx^2 + mx - n = 0$ are in arithmetic progression then $2l^3 - 9lm + 27n = 0$.
21. Solve the equation $x^3 + 4x^2 - 12x - 27 = 0$ given that its roots are in Geometric progression.
22. Use Descartes' Rule of Signs to determine the maximum number of positive and negative zeros of $x^5 + 2x^4 - x - 2 = 0$.

(8 × 2 = 16 Marks)

SECTION – C*Short essay type problems : Answer any SIX questions.*

23. Solve $(x^2 - 3y^2) dx + 2xy + dy = 0$.
24. Find the orthogonal trajectory of the family of curves $cx^2 + y^2 = 1$.
25. Solve $(D^2 - 8D + 12)y = 4e^{2x} + 5\sin(3x)$.
26. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 12 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1} .
27. Test for consistency and solve: $x + y + 2z = 4$, $2x - y + 3z = 9$, $3x - y - z = 2$.

28. Determine the eigen values and eigen vectors of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
29. Solve the equation $x^4 + 4x^3 - 5x^2 - 8x + 6 = 0$ having given that the sum of two roots is zero.
30. Solve $x^4 + 2x^2 - 16x + 77 = 0$ given that $-2 + i\sqrt{7}$ is a root.
31. Using bisection method find the root of the equation $x^3 + 4x^2 - 1 = 0$ to 3 places of decimal.

(6 × 4 = 24 Marks)

SECTION – D

Long essay type problems : Answer any TWO questions.

32. i). Solve $(D^2 - 2D + 1)y = xe^x$.
- ii). Solve $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 4y = 4x^2 - 6x^3$, $y(2) = 4$, $y'(2) = -1$.
33. i). Solve $\frac{dy}{dx} = \frac{y-x}{y-x+2}$.
- ii). Solve $(2x+5)^2 \frac{d^2y}{dx^2} - 6(2x+5) \frac{dy}{dx} + 8y = 6x$.
34. Show that $A = \begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable and find the diagonal form.
35. Approximate the real root to six decimal places of $x^3 + 5x - 3 = 0$ using Newton – Raphson method.

(2 × 15 = 30 Marks)
