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MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :……………..…**

Fifth Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Core Course: Mathematics – VII

AUMM544: Vector Analysis

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions. Each question carries **1** mark.*

1. Define gradient of a function $f(x, y, z)$.
2. Find the directional derivative of $f(x, y) = 4x^3y^2$ at $P(2,1)$ in the direction of $\boldsymbol{a} = 4\boldsymbol{i} - 3\boldsymbol{j}$.
3. Define a vector field.
4. If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, curl $\boldsymbol{r} =$ ------
5. Write Laplace's equation.
6. Define flow line to a vector field.
7. Find the work done by the force: $\boldsymbol{F} = xy\boldsymbol{i} + x^2\boldsymbol{j}$ on a particle that moves along the curve C whose equation is $x = y^2$ from (0, 0) to (1, 1).
8. Is $\boldsymbol{F} = y^2\boldsymbol{i} + 2xy\boldsymbol{j}$ conservative.
9. State Green's theorem in plane.
10. Let σ be a piecewise smooth oriented surface that is bounded by a simple closed piecewise smooth curve C with positive orientation. If the components of the vector field $\boldsymbol{F}$ are continuous and have continuous first partial derivatives on some open set containing σ, and if **T** is the unit tangent vector to C, then, $\oint_C \boldsymbol{F}.\boldsymbol{T}\, dS =$ _________.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions. Each question carries **2** marks.*

11. Sketch the gradient field of $\varphi(x, y) = x + y$.
12. Calculate the directional derivative of $f(x, y) = e^{xy}$ at $(-2,\ 0)$ in the direction of the unit vector making an angle $\pi/3$ with the positive x −axis.
13. Prove that $curl\ (\nabla\varphi) = 0$.
14. Prove that $\nabla\left(\frac{1}{r}\right) = \frac{\mathbf{r}}{r^3}$ where $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$
15. If $\boldsymbol{F} = 2x\boldsymbol{i} + \boldsymbol{j} + 4y\boldsymbol{k}$ and $\boldsymbol{G} = x\boldsymbol{i} + y\boldsymbol{j} - zy\boldsymbol{k}$, find $\nabla.(\boldsymbol{F} \times \boldsymbol{G})$.
16. Evaluate $\int_C \boldsymbol{F}.d\boldsymbol{r}$ where $\boldsymbol{F} = (x^2 + y^2)^{-3/2}(x\boldsymbol{i} + y\boldsymbol{j})$ and C is the curve whose vector equation is : $\boldsymbol{r}(t) = e^t \sin t\, \boldsymbol{i} + e^t \cos t\, \boldsymbol{j}$; $(0 \le t \le 1)$.
17. State the Fundamental Theorem of Work Integrals.
18. Evaluate $\oint_C (x^2 - y)dx + xdy$ where C is the circle $x^2 + y^2 = 4$.
19. Use Green's Theorem to find the work done by the force field $\boldsymbol{F} = xy\boldsymbol{i} + \left(\frac{x^2}{2} + xy\right)\boldsymbol{j}$ on a particle that starts at (5, 0), traverses the upper semicircle $x^2 + y^2 = 25$, and returns to the starting point along the x-axis.
20. Use a line integral to find the area enclosed by the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
21. Evaluate $\int_C \boldsymbol{F}.d\boldsymbol{r}$ where $\boldsymbol{F}(x, y) = (e^{-x} + 3y)\boldsymbol{i} + x\boldsymbol{j}$ and C is the boundary of the region between the circles $x^2 + y^2 = 16$ and $x^2 - 2x + y^2 = 3$.
22. Use divergence theorem to find the outward flux of the vector field $\boldsymbol{F}(x, y, z) = z\boldsymbol{k}$ across the sphere $x^2 + y^2 + z^2 = a^2$.

(8 × 2 = 16 Marks)

SECTION – C

*Answer any **SIX** questions. Each question carries **4** marks.*

23. Show that inverse square fields are conservative in any region that does not contain the origin.
24. Prove that $\text{div}\ (\varphi\mathbf{F}) = \varphi\ \text{div}\ \mathbf{F} + \nabla\varphi.\mathbf{F}$
25. A semicircular wire has the equation $y = \sqrt{25 - x^2}$ and density function $\delta(x, y) = 15 - y$. Find the mass of the wire.
26. Suppose a particle moves through a force field $\mathbf{F}(x, y) = xy\mathbf{i} + (x - y)\mathbf{j}$ from the point (0, 0) to the point (1, 0) along the curve: $x = t,\ y = \lambda t(1 - t)$. For what value of λ will the work done by the force field be one?
27. Show that $\mathbf{F}(x, y) = 2xy^3\mathbf{i} + (1 + 3x^2y^2)\mathbf{j}$ is conservative. Also find the scalar potential.

28. If $\mathbf{F}(x,y,z) = f(x,y,z)\boldsymbol{i} + g(x,y,z)\boldsymbol{j} + h(x,y,z)\boldsymbol{k}$ is a conservative field and f, g, h are continuous and have continuous partial derivatives, show that: $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$, $\frac{\partial f}{\partial z} = \frac{\partial h}{\partial x}$ and $\frac{\partial g}{\partial z} = \frac{\partial h}{\partial y}$.
29. Verify Green's Theorem for $\oint 3xydx + 2xydy$ along the rectangle bounded by $= -2,\ x = 4,\ y = 1,\ y = 2$.
30. Evaluate $\iint_\sigma y^2z^2dS$ where σ is the part of the cone $z = \sqrt{x^2 + y^2}$ that lies between $z = 1$ and $z = 2$.
31. Use Stoke's Theorem to evaluate $\oint_C \mathbf{F}.d\mathbf{r}$ where $\mathbf{F}(x,y,z) = z^2\boldsymbol{i} + 2x\boldsymbol{j} - y^3\boldsymbol{k}$ and C is the circle $x^2 + y^2 = 1$ in the xy-plane with counter clockwise orientation looking down the positive z-axis.

(6 × 4 = 24 Marks)

SECTION – D

*Answer any **TWO** questions. Each question carries **15** marks.*

32. Let $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $r = \|\boldsymbol{r}\|$, and let f be a differentiable function of one variable. If $\boldsymbol{F}(\boldsymbol{r}) = f(r)\boldsymbol{r}$, show that:
    i) $\nabla f(r) = \frac{f\prime(r)}{r}\boldsymbol{r}$
    ii) $div\ \boldsymbol{F} = 3f(r) + rf'(r)$
    iii) $curl\ \boldsymbol{F} = 0$
33. Let $\boldsymbol{F}(x,y) = e^y\boldsymbol{i} + xe^y\boldsymbol{j}$
    i) Verify that $\boldsymbol{F}$ is conservative in the entire xy-plane.
    ii) Find the potential for $\boldsymbol{F}$.
    iii) Find the work done by the field on a particle that moves from $(1,0)$ to $(-1,0)$ along the upper semi circular path.
34. i) Evaluate $\iint_\sigma (x + y + z)dS$ where σ is the surface of the cube defined by $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$.

    ii) Evaluate $\oint_C ydx - xdy$ where C is the cardioid: $r = a(1 + \cos\theta)$; and $0 \le \theta \le 2\pi$.
35. Verify Stoke's Theorem for the vector field $\boldsymbol{F}(x,y,z) = 2z\boldsymbol{i} + 3x\boldsymbol{j} + 5y\boldsymbol{k}$, taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$, for which $z \ge 0$ with upward orientation and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

(2 × 15 = 30 Marks)

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