# MAR IVANIOS COLLEGE (AUTONOMOUS) THIRUVANANTHAPURAM 

Reg. No. :
Name :
Fifth Semester B.Sc. Degree Examination, November 2016
First Degree Programme under CBCSS
Core Course: Mathematics - VI
AUMM543: Differential Equations
Time: $\mathbf{3}$ Hours
Max. Marks: $\mathbf{8 0}$

## SECTION - A

Answer ALL questions / problems in one or two sentences.

1. Write the general form of a second order linear differential equation.
2. Define an exact differential equation.
3. Find an exact differential equation whose solution is $u=x^{2}-y^{2}$
4. Find $A$ such that the following equation is exact. $\left(A x^{2} y+2 y^{2}\right) d x+\left(x^{3}+4 x y\right) d y=0$.
5. Find an integrating factor of $\frac{d y}{d x}+\frac{y}{x^{2}}=x^{2}$
6. Solve the equation $y^{\prime \prime}-y=0$
7. Define Euler Cauchy equation.
8. Is the functions $\mathrm{x}|\mathrm{x}|$ and $\mathrm{x}^{2}$ are linearly independent or dependent on $0 \leq x \leq 1$ ?
9. Find the wronskian $\mathrm{W}(\sin x, \cos x)$.
10. Solve $2 y^{\prime \prime}-9 y^{\prime}=0$.
( $10 \times 1=10$ Marks )

## SECTION - B

Answer any EIGHT questions / problems, not exceeding a paragraph.
11. Solve the initial value problem $x^{2} y^{\prime \prime}-2 x y^{\prime}+2 y=0, y(1)=3, y^{\prime}(1)=2$ if x and $x^{2}$ form a basis of solution.
12. Solve $y d x-x d y+\left(x^{2}+y^{2}\right) d x=0$

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13. Solve $2 y^{\prime \prime}+3 y^{\prime}+4 y=0$
14. Solve the initial value problem $\frac{d y}{d x}+2 y=e^{-x}$, given $y(o)=\frac{3}{4}$
15. Solve the initial value problem $y^{\prime \prime}+5 y^{\prime}+6 y=0$ given $y(0)=2$ and $y^{\prime}(0)=3$
16. Solve $\left(y-x^{2}\right) d x+\left(x^{2} \sin y-x\right) d y=0$.
17. Solve $2 x\left(y^{2}+1\right) d x+\left(x^{4}+1\right) d y=0$
18. Solve $(x-4) y^{4} d x-x^{3}\left(y^{2}-3\right) d y=0$
19. Show that $\cos (\mathrm{x}+\mathrm{y})$ is an integrating factor of $y d x+[y+\tan (x+y)] d y=0$ and solve.
20. Solve $\left(\mathrm{D}^{2}+2 \mathrm{D}+1\right) \mathrm{y}=0$.
21. Verify that $y_{p}=e^{-3 x}-3 e^{x}$ is a solution of the differential equation $y^{\prime \prime}-y=$ $8 e^{-3 x}$ and find a general solution.
22. Find the steady state oscillation of the mass spring system governed by the equation $y^{\prime \prime}+2 y^{\prime}+5 y=-13 \sin 3 t$.
( $8 \times 2=16$ Marks )

## SECTION - C

Short essay type problems : Answer any SIX questions.
23. Solve the initial value problem $\left(2 x \cos y+3 x^{2} y\right) d x+\left(x^{3}-x^{2} \sin y-y\right) d y=0$, given that $y(0)=2$
24. Find an integrating factor and solve $x^{2} y d x-\left(x^{3}+y^{3}\right) d y=0$.
25. Solve $\sec ^{2} x$ tanyd $x+\tan x \sec ^{2} y d y=0$
26. Solve $y e^{x} \sin y+\left(y e^{x} \cos y+1\right) \frac{d y}{d x}=0$
27. Solve $\frac{d y}{d x}+\frac{2 x+1}{x} y=e^{-2 x}$
28. A small body moves on a straight line so that its velocity equals the acceleration. If at $\mathrm{t}=0$ the body's distance from the origin is 2 meters and its velocity is 2 meters/sec. What is its distance and velocity at $\mathrm{t}=6 \mathrm{sec}$.
29. Solve $\frac{d^{2} y}{d x^{2}}+y=\operatorname{cosec} x$
30. Solve the initial value problem $y^{\prime \prime}-3 y^{\prime}-4 y=3 e^{2 x}$ given $\mathrm{y}(0)=1$ and $\mathrm{y}^{\prime}(0)=0$
31. Find a general solution of $\left(x^{2} D^{2}-4 x D+6\right) y=7 x^{4} \sin x$.
( $6 \times 4=24$ Marks $)$

## SECTION - D

Long essay type problems : Answer any TWO questions.
32. Find the general solution of the differential equation $\left(x^{2}+1\right) \frac{d^{2} y}{d x^{2}}-2 x \frac{d y}{d x}+2 y=$ $6\left(x^{2}+1\right)^{2}$, if $\mathrm{y}=\mathrm{x}$ is a solution of the corresponding homogeneous equation.
33. Using the method of variation of parameters, find the general solution of

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\frac{d^{2} y}{d x^{2}}+6 \frac{d y}{d x}+9 y=\frac{e^{-3 x}}{x^{3}}
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34. i) Solve $y^{2} d x+(3 x y-1) d y=0$
ii) Solve the initial value problem $(3 x+8)\left(y^{2}+4\right) d x-4 y\left(x^{2}+5 x+6\right) d y=0$ Given that $\mathrm{y}(1)=2$
35. Find the general solution of $\frac{d^{2} y}{d x^{2}}-3 \frac{d y}{d x}+2 y=2 x^{2}+e^{x}+2 x e^{x}+4 e^{3 x}$
( $\mathbf{2} \times \mathbf{1 5}=\mathbf{3 0}$ Marks )

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