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MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :.................... **Name :.....................**

Fourth Semester B.Sc. Degree Examination, June 2016

First Degree Programme under CBCSS

Complementary Course: Mathematics – IV (for Physics)

AUMM431.2d: Complex Analysis, Fourier Series and Fourier Transforms

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. What is the principal value of log(-5).
2. State whether the following is true or false:
   "If $f(z)$ is differentiable at z_0 , then it is continuous at z_0."
3. What is the value of the derivative of $\frac{(z-i)}{(z+i)} at\ z = i$
4. Evaluate $\int_{-i}^{i} \frac{dz}{z}$.
5. State Morera's theorem.
6. Find the residue of $f(z) = \frac{1}{z+z^2}$ at $z = 0$.
7. What is the fundamental period of $cos\pi x$.
8. Is $cos^3 x$ even or odd ?
9. What is the standard form of Fourier series for an odd periodic function of period $2L$.
10. Write down the formula for evaluating the Fourier cosine transform of an even function $f(x)$.

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems. Each question carries 2 marks.*

11. Is the function $f(z) = \bar{z} = x - iy$, differentiable. Justify your answer.

P.T.O.

12. Find all values of $(-8i)^{1/3}$.
13. Show that the function $f(z) = |z|^2$ is nowhere analytic.
14. Find the real and imaginary parts of the function $\cosh(x+iy)$.
15. Find the principal value of i^i
16. Evaluate $\int_C \bar{z}\,dz$ where C is the unit circle described counterclockwise.
17. Evaluate $\oint_C \frac{e^{2z}dz}{z^4}$ where C is the positively oriented unit circle $|z| = 1$.
18. Is the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ convergent or divergent ? Explain.
19. Find the Maclaurin series of $f(z) = cosz$.
20. Find the residue of $f(z) = \frac{5z}{(z+4)(z-1)^2}$ at the pole $z = 1$.
21. Find the Fourier series of $f(x) = \begin{cases} 1 \; if -\pi < x < 0 \\ -1 \; if \; 0 < x < \pi \end{cases}$.
22. Find the Fourier sine series of $f(x) = x; 0 < x < L$.

(8 × 2 = 16 Marks)

SECTION – C

*Answer any **SIX** questions. Each question carries 4 marks.*

23. Find an analytic function $f(z) = u + iv$ whose real part is $u = x^2 - y^2 - y$.
24. Show that if u is harmonic and v is a harmonic conjugate of u, then u is a harmonic conjugate of $-v$.
25. State Cauchy's integral Formula. Using Cauchy's integral formula evaluate $\oint_C \frac{dz}{z-3i}$ where C is the circle $|z| = \pi$ described counterclockwise.
26. Evaluate $\oint_C \frac{\tan z}{z^2-1}dz$ where C is the circle $|z| = \frac{3}{2}$ described counterclockwise.
27. i). State Cauchy's convergence principle for series.
    ii). Find the radius of convergence of the power series $\sum_{n=0}^{\infty}(n+2i)^n z^n$
28. State Laurent's theorem. Give the Laurent series expansions in powers of z for the function $f(z) = \frac{1}{(1-z)z^2}$ valid in $0 < |z| < 1$
29. Find the Fourier series of $f(x) = 3x^2; -1 < x < 1$

30. Find the Fourier sine series of $f(x) = \begin{cases} x \; if \; 0 < x < \frac{\pi}{2} \\ \pi - x \; \; if \; \frac{\pi}{2} < x < \pi \end{cases}$.

31. Show that the Fourier transform is a linear operator.

(6 × 4 = 24 Marks)

SECTION – D

*Answer any **TWO** questions. Each question carries 15 marks.*

32. i). Prove that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D then its component functions u and v are harmonic in D.

ii). Find an analytic function $f(z) = u + iv$ whose real part is $u = y^3 - 3x^2y$

iii). Determine a such that the function $u = e^{ax}cos5y$ is harmonic and find a harmonic conjugate.

33. i). State Cauchy's residue Theorem. Using Cauchy's residue theorem, evaluate the integral $\oint_C \frac{z\cosh \pi z}{z^4 + 13z^2 + 36} dz$ where C is the circle $|z| = \pi$ described counterclockwise.

ii). Evaluate $\oint_C \frac{z+1}{z^4 - 2z^3} dz$ where C is the circle $|z| = \frac{1}{2}$ described counterclockwise.

34. Find both the half range sine and cosine expansions of the function

$$f(x) = \begin{cases} \frac{2k}{L} x \; if \; 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) if \; \frac{L}{2} < x < L \end{cases}$$

35. i). Find the Fourier series for $f(x) = \frac{x^2}{4} in \; -\pi < x < \pi$ and deduce that $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots = \frac{\pi^2}{12}$.

ii). Find the Fourier transform of the function $f(x) = \begin{cases} k \; if \; 0 < x < a \\ 0 \quad otherwise \end{cases}$.

(2 × 15 = 30 Marks)

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