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# MAR IVANIOS COLLEGE (AUTONOMOUS)
# THIRUVANANTHAPURAM

**Reg. No. :…………………** **Name :………………….**

**Fifth Semester B.Sc. Degree Examination, November 2016**

**First Degree Programme under CBCSS**

**Core Course: Mathematics – V**

**AUMM542: Complex Analysis I**

Time: **3** Hours Max. Marks: **80**

## SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Define region in the complex plane.
2. Give an example of entire function.
3. State True / False: The function $f(z) = x^2 + iy^2$ is nowhere analytic.
4. Give the power series representation of $\sin z$ .
5. What do you mean by the radius of convergence of a power series ?
6. Express a nonzero complex number *z* in polar form.
7. If f is entire and if C is a smooth closed curve ,then $\int_C f(z)dz =$ ____________.
8. State Rectangle Theorem.
9. State True / False: The function $f(z) = x^2 + iv(x, y)$ is analytic for any choice of the real polynomial $v(x, y)$.
10. Show that $|e^z| = e^x$.

**(10 × 1 = 10 Marks)**

## SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Find the square roots of $-5 - 12i$.
12. Describe the set whose points satisfy the relation $|z - i| \leq 1$.Is it a region ?

P.T.O.

13. Show that $f(z) = \bar{z}$ is not differentiable at any point z.
14. Show that no non constant analytic polynomial can take imaginary values only.
15. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} nz^n$.
16. If $f = u + iv$ is analytic in a region D and u is constant, then show that f is constant.
17. Prove that $f(z) = |z|^2$ is not analytic at $z = 0$.
18. Find all solutions of $e^z = 1$.
19. Show that $\int_{-C} f = -\int_C f$, where $C$ is smooth curve.
20. Evaluate $\int_C f$ where $f(z) = x^2 + iy^2$ and C is given by $z(t) = t + it, 0 \leq t \leq 1$.
21. Show that the polynomial $P(x + iy) = 2xy + i(y^2 - x^2)$ is analytic.
22. Find the domain of convergence of $\sum_{n=0}^{\infty} n(z-1)^n$.

**(8 × 2 = 16 Marks)**

## SECTION – C

*Short essay type problems: Answer any **SIX** questions.*

23. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
24. Suppose $\sum_{n=0}^{\infty} C_n z^n$ is zero at all points of a nonzero sequence $\{z_k\}$ which converges to zero. Show that the power series is identically zero.
25. Find the radius of convergence of the following power series.

    a) $\sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$ b) $\sum_{n=0}^{\infty} z^{n!}$
26. If f is analytic in a region then prove that $|f|$ is constant implies f is constant.
27. Show that the power series $f(z) = 1 + z + \frac{z^2}{2!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots = \sum_{n=0}^{\infty} \frac{z^n}{n}$ is equal to $e^z$.
28. Find all analytic functions $f = u + iv$ with $u(x, y) = x^2 - y^2$.
29. Show that

$$f(z) = \begin{cases} \frac{xy(x+iy)}{x^2+y^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$ is not differentiable at orgin.

30. Suppose $G(t)$ is a continuous complex valued function of $t$. Then show that $\left|\int_a^b G(t)dt\right| \leq \int_a^b |G(t)|dt$.

31. If $C_1$ and $C_2$ are smoothly equivalent .Prove that

$$\int_{C_1} f = \int_{C_2} f$$

**(6 × 4 = 24 Marks)**

## SECTION – D

*Long essay type problems: Answer any **TWO** questions.*

32. i). Prove the following

a) $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$

b) $\bar{\bar{z}} = z$

c) $|z^2| = |z|^2$

d) $\overline{P(z)} = P(\bar{z})$ for any polynomial $P$ with real coefficients.

ii). Evaluate $\int_C z^k\, dz$ , $if\ k = -1\ and\ k \neq -1$ where $C: z = Re^{i\theta}, 0 \leq \theta \leq 2\pi$ .

33. i). Define power series

ii). Suppose $\overline{lim}\, |C_k|^{1/k} = L$ then show that

a) If $L = 0$, then $\sum_{n=0}^{\infty} C_k z^k$ converges for all $z$.

b) If $L = \infty$, then $\sum_{n=0}^{\infty} C_k z^k$ converges for $z = 0$ only.

c) If $0 < L < \infty$ , set $R = 1/L$ .then $\sum_{n=0}^{\infty} C_k z^k$ converges for $|z| < R$.

34. If $f = u + iv$ is differentiable at z then show that $f_x$ and $f_y$ exist and satisfy the Cauchy – Riemann equation $f_y = if_x$.Also show that $u_x = v_y$ and $u_y = -v_x$.

35. State and Prove Integral Theorem.

**(2 × 15 = 30 Marks)**

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