

MAR IVANIOS COLLEGE (AUTONOMOUS) THIRUVANANTHAPURAM

Reg. No. :....

Name :.....

Second Semester B.Sc. Degree Examination, June 2016 First Degree Programme under CBCSS Foundation Course – II: (for Mathematics) AUMM221: Foundations of Mathematics

Time: 3 Hours

Max. Marks: 80

SECTION – A

Answer ALL questions / problems in one or two sentences.

- 1. Find $12^{39} \mod (13)$.
- 2. Find the multiplicative identity in $\mathbb{Z}/m\mathbb{Z}$.
- 3. In $\mathbb{Z}/5\mathbb{Z} = \{[0], [1], [2], [3], [4]\}, \text{ find the inverses of } [2] \text{ and } [3].$
- 4. Find the intervals on which $f(x) = x^2 4x + 3$ is decreasing.
- 5. Find the point of inflection (if any) of $f(x) = (x a)^3$, where a is a constant.
- 6. Find all absolute extrema of $f(x) = x^3 3x^2 + 4$ on the interval $(-\infty, \infty)$.
- 7. Evaluate $\int \cos 5x \, dx$.
- 8. Find the displacement of the particle in the interval [0, 3] which is moving along a coordinate line so that its velocity at time t is $v(t) = t^2 2t$ m/s.
- 9. Find $\lim_{\theta \to 0} \frac{tan\theta}{\theta}$.
- 10. Find the eccentricity and the distance from the pole to the directrix of the conic

$$r=\frac{3}{2+\sin\theta}.$$

 $(10 \times 1 = 10 \text{ Marks})$

SECTION – B

Answer any **EIGHT** questions / problems, not exceeding a paragraph.

11. New Years Day fell on a Sunday in the year 2006. On what day of the week did New Years Day fall on in the year 2007 ?

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- 12. Define a complete set of representative for $\mathbb{Z}/m\mathbb{Z}$.
- 13. Find the set of units in $\mathbb{Z}/3\mathbb{Z}$.
- 14. Find the order of 2 modulo 7.
- 15. In $\mathbb{Z}/14\mathbb{Z}$, prove that $5^6 \equiv 1 \pmod{14}$.
- 16. If p is prime then prove that $\phi(p) = p 1$.
- 17. Locate the relative extrema of $f(x) = x^3 3x^2 + 3x 1$ if any.
- 18. Evaluate $\int \sin^2 x \cos x \, dx$.
- 19. Show that $\lim_{x\to 0} (1+x)^{1/x} = e$.
- 20. Evaluate $\int \frac{dx}{x^2\sqrt{4-x^2}} dx$.
- 21. Use cylindrical shell, find the volume of the solid generated when the region R in the first quadrant enclosed between y = x and $y = x^2$.
- 22. Find the polar coordinates of the point P whose rectangular coordinate is $(-2, 2\sqrt{3})$.

 $(8 \times 2 = 16 \text{ Marks})$

SECTION - C

Short essay type problems : Answer any SIX questions.

- 23. In $\mathbb{Z}/m\mathbb{Z}$, prove that [a] is a unit iff a and m are coprime.
- 24. Suppose $f(x) = x^4 + 5x^3 + 8x^2 + x + 15$, compute f(12) modulo 17.
- 25. Let B = 1194653. Let a = 2 and let $B = B_{13}$, then compute a^{B} .
- 26. Find the absolute maximum and minimum values of $f(x) = 2x^3 15x^2 + 36x$ on the interval (1, 5), and determine where these values occur.
- 27. Find $\frac{d}{dx}[\ln |\mathbf{x}|]$.
- 28. If f is continuous on a closed interval [a, b] then prove that there exists at least one number x* in [a, b] such that $\int_a^b f(x)dx = f(x^*)(b-a)$.
- 29. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, y = 2, and x = 0 is revolved about the y axis.
- 30. Find dy/dx if $y = (\ln x)^{tanx}$.
- 31. Evaluate $\int_0^1 tan^{-1}x \, dx$.

 $(6 \times 4 = 24 \text{ Marks})$

SECTION – D

Long essay type problems : Answer any TWO questions.

32. i). State and prove Fermat's theorem.
ii). If *e* is the order of *a* modulo *m*, and a^f = 1 (mod *m*), prove that *e* divides *f*.

- 33. i). Find the orders of the non-zero elements in $\mathbb{Z}/5\mathbb{Z}$.
 - ii). If $X = [5]_{16}$ is a solution of $[6]_{16} X = [14]_{16}$, find all other solutions.
 - iii). Sketch the graph of $r = \frac{6}{2 + \cos \theta}$ in polar coordinates.
- 34. i). Evaluate $\int_0^{+\infty} (1-x)e^{-x} dx$.
 - ii). Find the length of the curve $24xy = y^4 + 48$ from y = 2 to y = 4.
- 35. i). Find the area of the region in the first quadrant that is within the cardioid $r = 1 \cos\theta$.
 - ii). Express the polar equation $r = 2 + \cos(5\theta/2)$ parametrically, and generate the polar graph from the parametric equations using graphing utility.
 - iii). State Kepler's laws.

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 $(2 \times 15 = 30 \text{ Marks})$