# MAR IVANIOS COLLEGE (AUTONOMOUS) THIRUVANANTHAPURAM 

Reg. No. :
Name :

# Second Semester B.Sc. Degree Examination, June 2016 <br> First Degree Programme under CBCSS <br> Foundation Course - II: (for Mathematics) <br> <br> AUMM221: Foundations of Mathematics 

 <br> <br> AUMM221: Foundations of Mathematics}

Time: $\mathbf{3}$ Hours
Max. Marks: 80

## SECTION - A

Answer ALL questions / problems in one or two sentences.

1. Find $12^{39} \bmod (13)$.
2. Find the multiplicative identity in $\mathbb{Z} / m \mathbb{Z}$.
3. In $\mathbb{Z} / 5 \mathbb{Z}=\{[0],[1],[2],[3],[4]\}$, find the inverses of [2] and [3].
4. Find the intervals on which $f(x)=x^{2}-4 x+3$ is decreasing.
5. Find the point of inflection (if any) of $f(x)=(\mathrm{x}-\mathrm{a})^{3}$, where a is a constant.
6. Find all absolute extrema of $f(x)=x^{3}-3 x^{2}+4$ on the interval $(-\infty, \infty)$.
7. Evaluate $\int \cos 5 x d x$.
8. Find the displacement of the particle in the interval [ 0,3 ] which is moving along a coordinate line so that its velocity at time t is $\mathrm{v}(\mathrm{t})=\mathrm{t}^{2}-2 \mathrm{t} \mathrm{m} / \mathrm{s}$.
9. Find $\lim _{\theta \rightarrow 0} \frac{\tan \theta}{\theta}$.
10. Find the eccentricity and the distance from the pole to the directrix of the conic $r=\frac{3}{2+\sin \theta}$.

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\text { (10 } \times 1=10 \text { Marks) }
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## SECTION - B

Answer any EIGHT questions / problems, not exceeding a paragraph.
11. New Years Day fell on a Sunday in the year 2006. On what day of the week did New Years Day fall on in the year 2007?
12. Define a complete set of representative for $\mathbb{Z} / m \mathbb{Z}$.
13. Find the set of units in $\mathbb{Z} / 3 \mathbb{Z}$.
14. Find the order of 2 modulo 7 .
15. In $\mathbb{Z} / 14 \mathbb{Z}$, prove that $5^{6} \equiv 1(\bmod 14)$.
16. If p is prime then prove that $\phi(\mathrm{p})=\mathrm{p}-1$.
17. Locate the relative extrema of $f(x)=x^{3}-3 x^{2}+3 x-1$ if any.
18. Evaluate $\int \sin ^{2} x \cos x d x$.
19. Show that $\lim _{x \rightarrow 0}(1+x)^{1 / x}=\mathrm{e}$.
20. Evaluate $\int \frac{d x}{x^{2} \sqrt{4-x^{2}}} d x$.
21. Use cylindrical shell, find the volume of the solid generated when the region R in the first quadrant enclosed between $y=x$ and $y=x^{2}$.
22. Find the polar coordinates of the point P whose rectangular coordinate is $(-2,2 \sqrt{ } 3)$.

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(8 \times 2=16 \text { Marks })
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## SECTION - C

Short essay type problems : Answer any SIX questions.
23. In $\mathbb{Z} / \mathrm{mZ}$, prove that $[\mathrm{a}]$ is a unit iff a and m are coprime.
24. Suppose $f(x)=x^{4}+5 x^{3}+8 x^{2}+x+15$, compute $f(12)$ modulo 17 .
25. Let $\mathrm{B}=1194653$. Let $\mathrm{a}=2$ and let $\mathrm{B}=\mathrm{B}_{13}$, then compute $\mathrm{a}^{\mathrm{B}}$.
26. Find the absolute maximum and minimum values of $f(x)=2 x^{3}-15 x^{2}+36 x$ on the interval ( 1,5 ), and determine where these values occur.
27. Find $\frac{d}{d x}[\ln |\mathrm{x}|]$.
28. If f is continuous on a closed interval $[\mathrm{a}, \mathrm{b}]$ then prove that there exists at least one number $\mathrm{x}^{*}$ in $[\mathrm{a}, \mathrm{b}]$ such that $\int_{a}^{b} f(x) d x=f\left(x^{*}\right)(b-a)$.
29. Find the volume of the solid generated when the region enclosed by $y=\sqrt{ } x, y=2$, and $\mathrm{x}=0$ is revolved about the $\mathrm{y}-$ axis.
30. Find $d y / d x$ if $y=(\ln x)^{\tan x}$.
31. Evaluate $\int_{0}^{1} \tan ^{-1} x d x$.
( $6 \times 4=24$ Marks)

## SECTION - D

Long essay type problems : Answer any TWO questions.
32. i). State and prove Fermat's theorem.
ii). If $e$ is the order of $a$ modulo $m$, and $a^{f} \equiv 1(\bmod m)$, prove that $e$ divides $f$.
33. i). Find the orders of the non-zero elements in $\mathbb{Z} / 5 \mathbb{Z}$.
ii). If $\mathrm{X}=[5]_{16}$ is a solution of $[6]_{16} \mathrm{X}=[14]_{16}$, find all other solutions.
iii). Sketch the graph of $r=\frac{6}{2+\cos \theta}$ in polar coordinates.
34. i). Evaluate $\int_{0}^{+\infty}(1-x) e^{-x} d x$.
ii). Find the length of the curve $24 x y=y^{4}+48$ from $y=2$ to $y=4$.
35. i). Find the area of the region in the first quadrant that is within the cardioid $r=1-\cos \theta$.
ii). Express the polar equation $r=2+\cos (5 \theta / 2)$ parametrically, and generate the polar graph from the parametric equations using graphing utility.
iii). State Kepler's laws.
( $\mathbf{2} \times 15=30$ Marks )

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