MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :…… **Name : ……**

Third Semester B.A. Degree Examination, November 2015
First Degree Programme under CBCSS
Complementary Course: Mathematics – III (for Economics)
AUMM331.1a: Mathematics for Economics – III

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. What is $f(x)$, if $f'(x) = \frac{1}{x^3}$.
2. Find the antiderivative of $\frac{1}{\sqrt{x+1}}$.
3. Evaluate $\int_0^1 (x^2 + 5)\, dx$.
4. If $\int_{-1}^{2} f(x)\, dx = 7$ and $\int_{2}^{5} f(x)\, dx = 3$, find $\int_{-1}^{5} f(x) dx$.
5. Evaluate $\int_0^1 \sqrt{1-x}\, dx$.
6. Find the total cost function if it is known that the cost of zero output is c and that the marginal cost of an output x is ax + b.
7. Find the sum of the series : $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$
8. Determine whether the series $1+4+4^2+4^3+...$ is convergent or divergent.
9. The sum of n terms of a series is $\frac{n}{5n+1}$. Find the sum to infinity of the series.
10. Write the Taylor series expansion of e^x about $x = 0$.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Sketch the region whose area is represented by the integral $\int_1^5 3\,dx$ and hence evaluate it.
12. Evaluate $\int_{-1}^{2}(x^2 + e^x)\,dx$.
13. Find $\int \frac{2x+5}{\sqrt{x}}dx$.
14. Find $\int \frac{x}{(x^2-1)^2}\,dx$.
15. If the marginal cost function is $f'(q) = 2 + 3\sqrt{q} + \frac{5}{\sqrt{q}}$, find the total cost function when $f(1)=21$.
16. If Y is the constant stream of yield and r is the rate of interest, prove that the capitalization is given by $\frac{Y}{r}$.
17. Find the sum of the following infinite series:
    i). $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$
    ii). $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+...$
18. Expand $\ln(1 + x)$ about $x = 0$.
19. Sum to infinity the series $\sum_{n=1}^{\infty}\frac{1}{(n+1)!}$.
20. Find the Taylor series expansion of sin x, about $x = 0$.
21. Write the series whose sum of n terms is $\frac{n}{(n+1)}$. Find the sum to infinity of the series.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

22. Evaluate $1+\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+...$
23. Find the integrals:
    i). $\int x\ e^x\,dx$ ii). $\int \sqrt{5+2x}\,dx$

24. Find the integrals:

    i). $\int \frac{x^3}{2+x^4}\,dx$ ii. $\int \frac{-e^{\sqrt{x}}}{\sqrt{x}}\,dx$

25. Find the demand curve, if the elasticity of demand curve is x = (a – bp); a, b are constants.

26. If the marginal revenue function is $P_m = \frac{ab}{(x+b)^2} - c$, show that $p = \frac{a}{(x+b)} - c$ is the demand law.

27. Show that $\sum_{n=1}^{\infty} \frac{2n}{n!} = 2e$.

28. If $0 < x < 2$, prove that $\log(x^2) = 2\left[(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 + \ldots\right]$

29. Find the Taylor series expansion of log $(1 + \sin x)$, about $x = 0$.

30. Find the fraction corresponding to the repeating decimal $0.151515\ldots$ by expressing it as an infinite geometric series.

31. Find the total revenue at 18 units of a firm if its marginal revenue is $10 - 2x + x^2$.

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. Explain Domar's model of public debt and national income. Prove with usual notations that the ratio of debt to income approaches $\frac{a}{r}$.

33. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Simpson's rule by dividing the interval into four equal parts.

34. Find the sum of the series $\frac{1.3}{4.8} + \frac{1.3.5}{4.8.12} + \frac{1.3.5.7}{4.8.12.16} + \ldots$

35. The marginal cost function of a firm is given as MC $= \frac{a}{\sqrt{ax+b}}$. Find the total cost in terms of x if the cost of zero output is zero.

(2 × 15 = 30 Marks)

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