MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :………………….**

First Semester B.Sc. Degree Examination, November 2014
First Degree Programme under CBCSS
Core Course: Mathematics – I
AUMM141: Methods of Mathematics

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. State the principle of Mathematical induction.
2. Convert 101101 from binary to decimal.
3. Find the gcd of 657 and 963.
4. Use fundamental theorem of arithmetic to express 1000 as a product of primes.
5. Find $\lim_{h\to 0}\frac{(h+1)^{\frac{1}{3}}-1}{h}$.
6. Determine the continuity of $f(x)=\frac{1}{x-2}$ at $x = 2$.
7. Let $y = f(x)$ be a function of a real variable x defined in an open interval containing a. Define the continuity of f at a.
8. Find the distance from the vertex to the focus of the parabola $y = 2x^2$.
9. Find the equation of an ellipse centered at the origin with major axis of length 10 lying along the x – axis and minor axis of length 6 along the y – axis.
10. Find the eccentricity of the hyperbola $\frac{y^2}{25}-\frac{x^2}{144}=1$.

(10 x 1 = 10 Marks)

P.T.O.

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. State and prove Euclid's lemma.
12. Prove that if a prime number p divides a product of integers, then p divides one of the factors.
13. Explain Euclid's algorithm with an example.
14. Use congruences to show that 41 divides $2^{20} - 1$.
15. Prove that 11 divides a if and only if 11 divides the alternating sum of digits of a.
16. Show that $f(x) = \sin(x)$ is continuous for all values of x.
17. Find the value of the constant k that will make the function

    $f(x) = \begin{cases} kx^2 & , x \le 2 \\ 2x + k, & x > 2 \end{cases}$ continuous everywhere.
18. Find the equation of the line passing through the point (0, 4) and is parallel to the tangent of the curve $f(x) = \dfrac{x^2 + 8x}{2}$ at $x = -1$.
19. Use implicit differentiation to find $\dfrac{d^2y}{dx^2}$ if $4x^2 - 2y^2 = 9$.
20. Find the vertex of the parabola $x = y^2 + 4y + 5$.
21. Define an ellipse. State the reflection property of ellipses.
22. Write the general quadratic equation of a conic section. What are the conditions on its coefficients so that it becomes a parabola, ellipse and hyperbola ?

(8 x 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Prove that there are infinitely many primes.
24. Find integers x and y so that gcd (35, 91) = $35x + 91y$.
25. Find all solutions of the linear congruence $35x \equiv -14 \pmod{91}$.

26. Define Fermat numbers. For $m \neq n$, prove that $F(m)$ and $F(n)$ are relatively prime.

27. If p is a prime prove that $(p - 1)! \equiv -1 \pmod{p}$.

28. Find the tangent to the cycloid $x = a(\theta - \sin(\theta))$, $y = a(1 - \cos(\theta))$ at $\theta = \frac{\pi}{2}$.

29. Use implicit differentiation to show that the equation to the tangent line to the curve.

    $y^2 = kx$ at (x_0, y_0) is $y_0 y = \frac{1}{2}k(x + x_0)$.

30. Obtain the equation to an ellipse whose focus is (3,1), directrix $(x - y + 6) = 0$ and eccentricity $\frac{1}{2}$.

31. By a suitable rotation of the rectangular axes about the origin, remove the xy term in $5x^2 - 6xy + 5y^2 = 8$.

(6 x 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. (i). State Well Ordering principle. Use Well – Ordering principle to prove that if a and b are any positive integers, then there exists a positive integer n such that $na \geq b$.

    (ii). Use mathematical induction to prove that $1^2 + 2^2 + ... + n^2 = \frac{n(2n+1)(n+1)}{6}$.

33. (i). State and prove Fundamental theorem of arithmetic.

    (ii). Prove that the linear congruence $ax \equiv b \pmod{n}$ has a solution if and only if $d | b$ where $d = gcd(a, n)$. Also prove that if $d | b$, then it has d mutually incongruent solutions *mod n*.

34. (i). State and prove Intermediate value theorem.

    (ii). State and prove squeezing theorem.

    (iii). Use squeezing theorem to prove that $\lim_{x \to 0} x \sin \frac{1}{x} = 0$.

35. (i). Find the center, foci, vertices and directrices of the ellipse

$$9x^2 + 25y^2 - 18x - 100y - 116 = 0$$

(ii). A wheel of radius a rolls along a horizontal straight line. Find the parametric equations for the path traced by a point on the wheel's circumference.

(2 x 15 = 30 Marks)

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