MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :………………….**

Fourth Semester B.Sc. Degree Examination, June 2016

First Degree Programme under CBCSS

Complementary Course: Mathematics – IV (for Chemistry)

AUMM431.2b: Abstract Algebra and Linear Transformations

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Give an example of a non−abelian group.
2. Find the quotient and the remainder obtained when –38 is divided by 7.
3. Give an example of an infinite cyclic group.
4. Define a skew field.
5. Determine an element $b \in Z$ *such that* $a * b = a$ *where* $*$ *is defined by* $a * b = a + b - 2$.
6. What is the geometrical interpretation of the statement "two vectors in R^2 are linearly dependent" ?
7. Define a linear transformation.
8. Let $T: R^5 \rightarrow R^3$ be a linear transformation. What is the order of the standard matrix corresponding to T ?
9. Write the standard matrix corresponding to the transformation reflection through origin.
10. Say True or False: "A linear transformation T is 1 − 1 if and only columns of matrix corresponding to T is not linearly dependent".

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. State True or False:
    i). Every abelian group is cyclic.

ii). Set of all natural numbers under addition is a group.

iii). Every field is a ring.

iv). Intersection of any two subgroup of a group is again a subgroup.

12. Compute the subgroups generated by <2> and <3> of $< Z_8, +_8 >$.
13. Find all orders of the subgroups $< Z_{12}, +_{12} >$ and $< Z_8, +_8 >$.
14. Define order of an element in a group. Find the order of 2 in the group $< Z_6, +_6 >$.
15. Justify the statement "Set of all integers under usual addition and multiplication forms a field".
16. Find the number of elements of the cyclic subgroup generated by 25 in $< Z_{30}, +_{30} >$.
17. Define automorphism. Find the number of automorphisms of $< Z_6, +_6 >$.
18. Give a necessary and sufficient condition for any subset of a group is to be a subgroup.
19. Is the transformation defined by $T(x, y) = (5y, x)$linear. Justify ?
20. Find the standard matrix A for the dilation transformation T(X)=3X for any $X \in R^2$.
21. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ does the transformation $X \to A$X projects points in R^3 to $X_1X_2 -$ plane.
22. Let $T(x_{1,}x_2) = (x_1 + x_{2,}3x_1 + 2x_2)$.Find an $X \in R^2$ such that T(X) = (3, 5).

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Consider the binary operation * on the set of positive rational numbers Q^+ defined by a*b= $ab/4$. Show that < Q^+,$*$> is a group.
24. Show that in a group G, if $(a * b)^2 = a^2 * b^2 \; \forall a, b \in G$, then G is abelian.
25. Draw a sub graph diagram for Klien – 4 group.
26. Find all generators of the cyclic group Z_8.
27. Show that the vectors (1, –1,2);(2,4,2) and(2,1,0) are linearly independent.
28. Let $T: R^2 \to R^3$be a linear transformation defined by
    $T(x, y) = (3x + y,\ 5x + 7y, x + 3y)$. Show that T is one – one. Is T onto. Justify.

29. Let $T(e_1) = \begin{bmatrix} -5 \\ -7 \\ 2 \end{bmatrix}$ and $T(e_2) = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}$ where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find a formula for the image of an arbitrary vector in R^2 under T.

30. Determine whether the columns of the matrix $A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$ form a linearly independent set.

31. i). If {u,v,w} is a linearly independent set then show that the set {u+v, v+w, w+u} is also linearly independent.

    ii). If $S = \{v_1, v_{2,} v_3, v_4\}$ such that $v_4 = 0$.Is S linearly independent. Justify.

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. i). Give all subgroups of Z_{18} and draw the sub graph diagram.

    ii). Define Ring and a Field.

    iii).Give examples of:

    a). Ring with unity

    b). Ring without unity

    c). Ring but not a Field

33. i). Find all Units of Z_{14} and Z.

    ii). Construct a non abelian group of order 6.

34. i). Find the value of ‘a’ such that $\begin{bmatrix} 1 \\ 2a \end{bmatrix}$ *and* $\begin{bmatrix} a \\ a+2 \end{bmatrix}$ *is* linearly dependent.

    ii). Let T be a linear transformation defined by T(X) = AX. find a vector X such whose image under T is b and check whether X is unique, where

    $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}$ and b = $\begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$.

35. i). Let A = $\begin{bmatrix} -9 & -4 & -9 & 4 \\ 5 & -8 & -7 & 6 \\ 7 & 11 & 16 & -9 \\ 9 & -7 & -4 & 5 \end{bmatrix}$. If T is linear transformation defined by T(X) = AX. find all X such that TX=0.

    ii). If T:$R^2 \to R^2$ is linear transformation defined by T(X) =AX. Find the image of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ under T, where A is given by $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(2 × 15 = 30 Marks)

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