MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :…………………   Name :………………….

First Semester B.Sc. Degree Examination, November 2015
First Degree Programme under CBCSS
Core Course: Mathematics – I
AUMM141: Methods of Mathematics
(for 2015 Admissions Only)

Time: **3** Hours   Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. State whether the statement, " for all $n \geq 1$, $2^n > 2 + n$ " is true or false. Give reason.

2. State fundamental theorem of arithmetic.

3. Write 9987 in base 100.

4. If a divides bc and $(a, b) = 1$, then prove that a divides c.

5. Suppose that a car moves with a constant velocity 88 ft/s in the positive direction of an s – axis. Given that the s co – ordinate of the car at time $t = 0$ is $s = 100$, find an equation for s as a function of t.

6. Draw the graph of the parametric curve $x = 2t - 3$, $y = 6t - 7$ by eliminating the parameter.

7. Consider the function $f(x) = 6x - x^2$ and the point $P(2, 8)$. Find the slope of secant lines to the graph of $y = f(x)$ determined by P and the point on the graph at $x = 3$.

8. Find $\dfrac{d^2 y}{dx^2}$ for $x^2 - 3y^2 = 19$.

9. Find an equation of the parabola that is symmetric about the y – axis has its vertex at the origin and passes through the point (5, 2).

10. State the reflection property of parabolas.

(10 × 1 = 10 Marks)
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SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. For all $n \geq 1$, prove that $1 + 3 + ... + (2n - 1) = n^2$.

12. If $b = aq + r$, then prove that $(b, a) = (a, r)$.

13. Given integers a, b, e, there are integers m and n with $am + bn = e$ if and only if (a, b) divide e.

14. Prove that $6 \cdot 4^n \equiv 6 \pmod 9$ for any $n \geq 0$.

15. Sketch the graph of the function.
$$f(x) = \begin{cases} 0 & x \leq -1 \\ \sqrt{1 - x^2} & -1 < x < 1. \\ x & x \geq 1 \end{cases}$$

16. Explain why a linear function is appropriate for the data in the table

x	1.5	2.5	3.5	5.5	9.5
y	0.3	1.1	1.9	3.5	6.7

. Find a linear equation that relates x and y.

17. Show that $|x|$ is continuous everywhere.

18. Prove that $\lim_{x \to 0} \dfrac{\sin x}{x} = 0$.

19. If n is a positive integer, then $\dfrac{dx^n}{dx} = nx^{n-1}$.

20. Find $\dfrac{d^2 y}{dx^2}$ of $4x^2 - 2y^2 = 9$.

21. Sketch the graph of the parabolas $x^2 = 12y$ and $y^2 + 8x = 0$.

22. Find an equation of the ellipse with foci $(0, \pm 2)$ and major axis with end points $(0, \pm 4)$.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Write 1987 in base 1000, 8, 2.

24. Find the greatest common divisor of 17017 and 18900.

25. If $n > 4$ is not prime, then prove that $(n - 1)! \equiv 0 \pmod n$.

26. State the vertical line test and explain it through an example.

27. Sketch the graph of $y = \sqrt{x-3}$, $y = |x-3|$, $y = x^2 - 4x + 5$.

28. State the intermediate value Theorem and prove that If f is continuous on $[a, b]$ and if $f(a)$ and $f(b)$ are nonzero and have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b).

29. Draw the graph of $y = |x|$ and prove that it is not differentiable at $x = 0$. Find a formula for $f'(x)$.

30. Sketch the graphs of the ellipses $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ and $x^2 + 2y^2 = 4$ showing their foci each.

31. Identify and sketch the graph of $153x^2 - 192xy + 97y^2 - 30x - 40y - 200 = 0$.

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. Prove that any non empty set of natural numbers has a least element.

33. Define the terms limit, continuity, derivative of a function and instantaneous velocity of a particle. Suppose a ball is thrown vertically upward and the height in feet of the ball t seconds after it release is modeled by the function,
$s(t) = -16t^2 + 29t + 6$; $0 \le t \le 2$. What is a reasonable estimate for the instantaneous velocity of the ball at time $t = 0.5s$.

34. State and prove the product and quotient rule for differentiation. Also find the slopes of the curve $y^2 - x + 1 = 0$ at the points $(2, -1)$ and $(2, 1)$.

35. Sketch the graphs of the hyperbolas $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$ and $y^2 - x^2 = 1$, showing their vertices, foci and asymptotes.

(2 × 15 = 30 Marks)

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