



MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, November 2015

First Degree Programme under CBCSS

Complementary Course: Mathematics – I (for Chemistry)

AUMM131.2b: Differentiation and Analytic Geometry

(for 2015 Admissions Only)

Time: 3 Hours

Max. Marks: 80

SECTION – A

Answer ALL questions / problems in one or two sentences.

1. For what values of x when $y = 0$, if $y = x^2 - 6x + 8$.
2. Find the natural domain of $f(x) = \tan x$.
3. Find $\lim_{x \rightarrow 5} (x^2 - 4x + 3)$.
4. Find the average rate of change of $y = x^2 + 2x + 1$ with respect to x over the interval $[3, 5]$.
5. Define absolute minimum of a function in an interval I .
6. State Rolle's theorem.
7. Evaluate $\lim_{(x,y) \rightarrow (1,4)} [5x^3y^2 - 9]$.
8. State the extreme value theorem.
9. Find the rectangular coordinates of the point P whose polar coordinates are $(6, \frac{2\pi}{3})$.
10. State the Reflection Property of Ellipses.

(10 × 1 = 10 Marks)

SECTION – B

Answer any EIGHT questions / problems, not exceeding a paragraph.

11. Find the parametric equations for the portion of the parabola $x = y^2$ joining $(1, -1)$ and $(1, 1)$, oriented down to up.

P.T.O.

12. Define continuity of a function f at $x = c$.
13. Compute $\lim_{x \rightarrow 1} \frac{\sin(\pi x)}{x-1}$.
14. Find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$.
15. Locate the relative extrema of $f(x) = x^3 - 3x^2 + 3x - 1$, if any.
16. Find the slope of the curve $y = x^2 + 1$ at the point $(2, 5)$ and use it to find the equation of the tangent at $x = 2$.
17. Find the velocity and acceleration of a particle which moves on a parabola $s(t) = 16t^2 - 29t + 6$ at $t = 3$.
18. Find $\frac{d}{dx} [\ln(\tanh x)]$.
19. Determine $f_x(1, 3)$ for the function $f(x, y) = 2x^3y^2 + 2y + 4x$.
20. Show that when f is differentiable, a function of the form $z = f(xy)$ satisfies the equation $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$.
21. Find the equation of the parabola with focus $(0, -3)$ and directrix $y = 3$.
22. Sketch the graph of the curve $r = 1$ in polar coordinates.

(8 × 2 = 16 Marks)

SECTION – C

Short essay type problems : Answer any SIX questions.

23. Sketch the graph of the parametric equations. $x = \cos t$ and $y = \sin t$, $(0 \leq t \leq \pi)$.
24. Show that $|x|$ is continuous everywhere.
25. Prove that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.
26. A liquid form of penicillin manufactured by a pharmaceutical firm is sold in bulk at a price of Rs.200/- per unit. If the total production cost (in rupees) for x units is $C(x) = 5000000 + 80x + 0.003x^2$ and if the production capacity of the firm is at most 30,000 units in a specified time, how many units of penicillin must be manufactured and sold in that time to maximize the profit ?
27. Find the interval $[a, b]$ on which $f(x) = x^4 + x^3 - x^2 + x - 2$ satisfies the Rolle's theorem.
28. Find $\frac{d}{dx} [\ln |x|]$.

29. Let $u = (x^2 + y^2 + z^2)^{-1/2}$. Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.
30. Describe the graph of the function $x^2 - y^2 - 4x + 8y - 21 = 0$.
31. Find the equation of the ellipse with foci $(0, \pm 2)$ and major axis with end points $(0, \pm 4)$.

(6 × 4 = 24 Marks)

SECTION – D*Long essay type problems : Answer any TWO questions.*

32. i). Sketch the graph of $y = 2 - \frac{1}{x+1}$ by transforming the graph of $y = \frac{1}{x}$ appropriately.
- ii). Evaluate the following limits.
- (a). $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$. (b). $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$
33. i). Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.
- ii). Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$.
34. i). Let $f(x, y) = x^2y + 5y^3$. Find the slope of the surface $z = f(x, y)$ in the x – direction at the point $(1, -2)$.
- ii). Verify Euler's theorem for the function $f(x, y) = (x^2 + xy + y^2)^{-1}$.
- iii). Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3 , and requiring the least amount of material for its construction.
35. i). Describe the graph of the function $x^2 - y^2 - 4x + 8y - 21 = 0$.
- ii). Find the new coordinates of the point $(2, 4)$ if the coordinate axes are rotated through an angle of $\theta = 30^\circ$.
- iii). For the conic $r = \frac{3}{2 - 2\cos \theta}$ find the eccentricity and distance from the pole to the directrix and sketch the graph.

(2 × 15 = 30 Marks)
