MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :………………….**

Third Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Complementary Course: Mathematics – III (for Chemistry)

AUMM331.2b: Vector Analysis and Theory of Equations

(*for 2014 Admissions – **Improvement** Only*)

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. State the fundamental theorem of algebra.
2. Form a rational cubic equation whose roots are 2, 1 + 2i.
3. Give an example of an equation for which $\alpha = 1$ and $\beta = 2$ are double roots.
4. Find two numbers *a* and *b* such that a real root of $f(x) = x^3 - x - 1 = 0$ lies between *a* and *b*.
5. Evaluate $\int_0^{\pi} \{ (\cos t)\,\boldsymbol{i} + \boldsymbol{j} - 2t\boldsymbol{k} \}\, dt$.
6. Find the value of a if $\mathbf{F} = (axy - z^2)\,\mathbf{i} + (x^2 + 2yz)\,\mathbf{j} + (y^2 - axz)\,\mathbf{k}$ is irrotational.
7. Prove that the vector $\mathbf{F} = x(y - z)\,\mathbf{i} + y(z - x)\,\mathbf{j} + z(x - y)\,\mathbf{k}$ is solenoidal.
8. State Green's theorem.
9. Find the arc length parameterization of the cylinder $x^2 + (y - 3)^2 = 9,\ 0 \le z \le 5$.
10. Define irrotational vector field.

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. Solve the equation $x^4 + x^3 - 33x^2 + 61x - 14 = 0$, given that $2 + \sqrt{3}$ is a root.
12. State Descartes' rule of signs and apply it to prove that the equation $5x^3 + 2x + 6 = 0$ have one negative and two imaginary roots.

13. If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetic progression, show that $2p^3 - 9pq + 27r = 0$.
14. Find the curl of the vector $xyz\,\mathbf{i} + (xz^2 - y^2z)\,\mathbf{j} + (xz^2 - y^2z)\mathbf{k}$ at $(1, 2, -1)$.
15. If $\mathbf{a}$ is a constant vector and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, show that $\nabla \times (\mathbf{a} \times \mathbf{r}) = 2\mathbf{a}$.
16. Evaluate $\int_C (x - y + z - 2)ds$ where C is the straight line segment from (0, 1, 1) to (1, 0, 1).
17. Find the work done by the conservative field $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} = \nabla xyz$ along any smooth curve C joining the point A (1, 3, – 6) and (3, 3, 5).
18. If $r = xi + yj + zk$ then prove that i). div r = 3 ii). curl r = 0.
19. Find parametric equation of the line tangent to $\mathbf{r}(t) = e^{2t}\,\boldsymbol{i} - 2\sin 5t\,\mathbf{j}$ at t = 0.
20. Find the length of the indicated portion of the curve $\mathbf{r}(t) = 6t^3\mathbf{i} - 2t^3\mathbf{j} - 3t^3\mathbf{k}, 1 \le t \le 2$.
21. Find $\nabla.(F \times G)$ where $\mathbf{F}(x,y,z) = 2x\mathbf{i} + \mathbf{j} + 5y\mathbf{k}$ and $\mathbf{G}(x,y,z) = x\mathbf{i} + y\mathbf{j} - z\mathbf{k}$.
22. Use Green's theorem to find the area of the region enclosed by the curve $\mathbf{r}(t) = (a \cos t)\,\mathbf{i} + (b \sin t)\,\mathbf{j}, 0 \le t \le 2\pi$.

(8 × 2 = 16 Marks)

SECTION – C

*Short essay type problems : Answer any **SIX** questions.*

23. Solve the equation $24x^3 - 14x^2 - 63x + 45 = 0$, one root being double the other.
24. Find to four places of decimals a real root of $x^3 - 3x + 1 = 0$ which lies between 1 and 2.
25. Find the directional derivative of the function $f = xy^2z - x^2yz^3$ at the point (–1, 2, 1) in the direction 3i + j – 4k.
26. Find the curvature for the helix $\mathbf{r}(t) = (a \cos t)\mathbf{i} + (b \sin t)\mathbf{j} + bt\mathbf{k}, a, b \ge 0, a^2 + b^2 \ne 0$.
27. Verify that the field $\mathbf{F}(x,y) = 2xy^3\mathbf{i} + 3x^2y^2\mathbf{j}$ is conservative. Find a potential function φ for the field.
28. Find the work done in moving a particle in the force field $\mathbf{f} = 3x^2\,\mathbf{i} + (2xz - y)\,\mathbf{j} + z\,\mathbf{k}$ along the space curve $x = 2t^2$, $y = t^2$, $z = 4t^2 - 1$ from $t = 0$ to $t = 1$.
29. Evaluate the integral $\int_C (xy + z^3)\,ds$ from (1, 0, 0) to (–1, 0, π) along the curve $x = \cos t, y = \sin t, z = t, 0 \le t \le \pi$.
30. Verify Green's theorem for the field $F(x,y) = (x - y)\,i + x\,j$ and the region bounded by the unit circle C: $r(t) = (\cos t)\,i + (\sin t)\,j, 0 \le t \le 2\pi$.

31. Use stokes theorem to evaluate $\oint_C \boldsymbol{F}.d\boldsymbol{r}$ for the hemisphere $\sigma : x^2 + y^2 + z^2 = 9, z \geq 0$, its bounding circle $x^2 + y^2 = 9$, $z = 0$, and the field $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$.

(6 × 4 = 24 Marks)

SECTION – D

Long essay type problems : Answer any ***TWO*** *questions.*

32. Solve the equation $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$, given that the roots are in geometric progression.
33. Obtain a root to 4 decimal places of $x^5 + 5x + 1 = 0$ by using Newton – Raphson method.
34. The position function of a particle is given by $\mathbf{r} = e^t \cos t\, \mathbf{i} + e^t \sin t\, \mathbf{j}$. Evaluate the scalar tangential and normal as well as vector tangential and normal component of acceleration at $t = \frac{\pi}{4}$. Also find the curvature of the path at the point where the particle is situated at $t = \frac{\pi}{4}$.
35. Use the Divergence theorem to find the outward flux of the vector field $\mathbf{F}(x,y,z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ across the surface of the region that is enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane z = 0.

(2 × 15 = 30 Marks)

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