



Board of Studies in Mathematics

MAR IVANIOS COLLEGE, THIRUVANANTHAPURAM
(An Autonomous College, affiliated to the University of Kerala)

Syllabus for M.Sc. Mathematics
2017 Admission

M.Sc. Mathematics Course Structure and Mark Distribution

Semester	Paper Code	Title of Paper	Instructional Hours per week	Duration of ESE	Maximum Marks		
					CE	ESE	Total
I	APMM121	Linear Algebra	6	3hours	25	75	100
	APMM122	Real Analysis-I	6	3hours	25	75	100
	APMM123	Differential Equation	6	3hours	25	75	100
	APMM124	Topology-I	7	3hours	25	75	100
II	APMM221	Abstract Algebra	6	3hours	25	75	100
	APMM222	Real Analysis-II	6	3hours	25	75	100
	APMM223	Topology -II	7	3hours	25	75	100
	APMM224	Scientific Programming with Python	6	3hours	25	75	100
III	APMM321	Complex Analysis-I	6	3hours	25	75	100
	APMM322	Functional Analysis-I	6	3hours	25	75	100
	APMM323	Elective -1	6	3hours	25	75	100
	APMM324	Elective -2	6	3hours	25	75	100
IV	APMM421	Complex Analysis-II	6	3hours	25	75	100
	APMM422	Functional Analysis-II	6	3hours	25	75	100
	APMM423	Elective -3	6	3hours	25	75	100
	APMM424	Elective -4	6	3hours	25	75	100
	APMM425	Dissertation / Project				80+20	100
		Comprehensive Viva					100
	Grand Total						1800

Elective Courses

Elective Course-1

Semester	Course Code	Course Title
III	APMM 323	Automata Theory
III	APMM 323	Approximation Theory
III	APMM 323	Operations Research
III	APMM 323	Probability and Statistics

Elective Course -2

Semester	Course Code	Course Title
III	APMM 324	Field Theory
III	APMM 324	Graph Theory
III	APMM 324	Mechanics
III	APMM 324	Theory of Wavelets

Elective Course -3

Semester	Course Code	Course Title
IV	APMM 423	Coding Theory
IV	APMM 423	Differential Geometry
IV	APMM 423	Geometry of Numbers
IV	APMM 423	Advanced Graph Theory

Elective Course -4

Semester	Course Code	Course Title
IV	APMM 424	Analytic Number Theory
IV	APMM 424	Category Theory
IV	APMM 424	Commutative Algebra
IV	APMM 424	Representation theory of Finite Groups

APMM 121 LINEAR ALGEBRA

Text: Sheldon Axler, “Linear Algebra Done Right” 2nd Edition, Springer

UNIT I

Vector spaces: Definition, Examples and properties, Subspaces. Sum and Direct sum of subspaces, Span and linear independence of vectors, Definition of finite dimensional vector spaces, Bases: Definition and existence, Dimension Theorems.

[Chapters 1, 2]

UNIT II

Linear maps, their null spaces and ranges, Operations on linear maps in the set of all linear maps from one space to another, Rank-Nullity Theorem, An application of rank-nullity theorem in the theory of linear equations, Matrix of linear map, Invertibility of linear maps, isomorphism of vector spaces.

[Chapter 3]

UNIT III

Invariant subspaces, Definition of eigen values and vectors, Polynomials of operators, Upper triangular matrices of linear operators, Equivalent condition for a set of vectors to give an upper triangular operator, Diagonal matrices, Invariant subspaces on real vector spaces.

[Chapter 5]

UNIT IV

Operators on Complex vector spaces: Concept of generalized eigen vectors, Nilpotent operators, characteristic polynomial of an operator, Cayley-Hamilton theorem for operators on complex spaces, Condition for an operator to have a basis consisting of generalized eigenvectors, Minimal polynomial. Jordan form of an operator

Operators on Real vector spaces: Eigen values of square matrices, block upper triangular matrices, The Characteristic polynomial, Cayley-Hamilton Theorem for operators on real spaces.

(Comparison with operators on real vector spaces may be made based on the results in Chapter-9. Proof of other theorems in Chapter-9 may be omitted. General case of Cayley-Hamilton Theorem may be briefly sketched from the reference text. This is not meant for end semester examination)

[Chapters 8 & 9]

UNIT V

Change of basis, trace of an operator, showing that trace of an operator is equal to the trace of its matrix, determinant of an operator, invertibility of an operator and its determinant, relation between characteristic polynomial and determinant, determinant of matrices of an operator w.r.t. two bases are the same. Determinant of a matrix (The section, volumes may be omitted)

[Chapter 10]

References

1. Kenneth Hoffman and Ray Kunze, “Linear Algebra”, Prentice Hall, 1981.
2. I. N Herstein, “Linear Algebra”, Wiley Eastern.
3. S. Kumaresan, “Linear Algebra”, Prentice Hal, 2000.

UNIT IV

Multivariate Calculus: Sequences, continuity and limits.

Sequences in \mathbb{R}^2 , Sub-sequences and Cauchy sequences, Compositions of continuous functions, Piecing continuous functions on overlapping subsets, Characterizations of continuity, Continuity and boundedness, Continuity and convexity, Continuity and intermediate value property, Uniform continuity, Implicit function Theorem, Limits and continuity.

[Chapter 2: Sections 2.1, 2.2 (excluding Continuity and monotonicity, Continuity, Bounded Variation, Bounded Bivariation), 2.3 (Excluding Limits from a quadrant, Approaching Infinity) of Text 2]

UNIT V

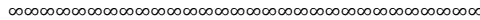
Partial and Total Differentiation.

Partial derivative, Directional derivatives, Higher order partial derivatives, Higher order directional derivatives, Differentiability, Taylor's Theorem and Chain rule, Functions of three variables, Extensions and analogues, Tangent planes normal lines to surfaces.

[Chapter 3 (excluding Section 3.4 and last subsection of Section 3.5) of Text 2]

References

1. J.A Dieudonne, *Foundations of Modern Analysis*, Academic Press.
2. W. Rudin, *Real and Complex analysis*, Tata Mc-Graw Hill.
3. Tom M Apostol, *Calculus*, Volume-1, Wiley Edition.
4. Tom M Apostol, *Calculus*, Volume-2, Wiley Edition.



APMM 123 Differential Equations

Texts: (1) George F Simmons, *Differential Equations with Applications and Historical Notes*, Tata McGraw Hill.

(2) T. Amarnath, *An Elementary Course in Partial Differential Equations*, Narosa.

UNIT 1

Solving Second Order Linear Equations – The Method of Undetermined Coefficients, The Method of Variation of Parameters, The Method of Successive Approximations and Picard’s Theorem.
(Chapter-3: Sections 18, 19; Chapter-11: Sections 55, 56, 57 of Text 1)

UNIT 2

Series solutions of first order differential equations - ordinary points, regular singular points, Gauss’s hypergeometric equations, the point at infinity, Chebyshev polynomials.
(Chapter-5: Sections 25-31 and Appendix D, excluding the minimax property of Text 1)

UNIT 3

Special functions- Legendre polynomials, Bessel functions, The gamma function
(Chapter-6: Sections 32-35 of Text 1)

UNIT 4

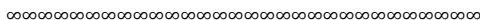
First order partial differential equations: Curves and Surfaces, Genesis of first order partial differential equations, Classification of integrals, Linear equations of first order, Pfaffian differential equations, Compatible systems, Charpit’s equations, Jacobi’s method.
(Chapter-1: Sections 1.1 to 1.8 of Text 2)

UNIT 5

Second order partial differential equations: Classification of Second order partial differential equations, One dimensional wave equation, Vibrations of finite string, Vibrations of a semi-infinite string, Vibrations of an infinite string, Laplace’s equation, Boundary value problems, Maximum and minimum principles.
(Chapter-2: Sections 2.1, 2.2, 2.3.1 to 2.3.3 and 2.4.1 to 2.4.2 of Text 2)

References:

1. Ian Sneddon, *Elements of Partial Differential Equations*, McGraw Hill, New York, 1957.
2. Phoolan Prasad, Renuka Raveendran, *Partial Differential Equations*, Wiley, Eastern
3. Zahir Ahsan, *Differential Equations and their Applications*, Prentice Hall, 1999
4. Coddington and Levinson, *Theory of Ordinary Differential Equations*, McGraw Hill, New York, 1955.
5. G. Birkoff and G. C Rota, *Ordinary Differential Equations*, Wiley and sons – 3rd edn, 1978.
6. M. Ram Mohan Rao, *Ordinary Differential Equations and Applications*



APMM 124 TOPOLOGY-I

Text: Principles of Topology by Fred H Croom, Baba Barkha Nath Printers (India), Third Reprint 2009.

In this Course, we discuss the basics of Topology based on Chapters 3 and 6 of the Text. Students may be motivated as discussed in the first two chapters of the text.

UNIT I

Metric Spaces: Definition, Examples, Open Sets, Closed Sets, Interior, Closure and Boundary
(Sections: 3.1, 3.2 and 3.3)

UNIT II

Continuous Functions, Equivalence of metric spaces, Complete metric spaces – Cantor’s Intersection Theorem.
(Sections: 3.4, 3.5 and 3.7 (Exercise may be included 3.7(3))

UNIT III

Topological Spaces: Definition, Examples, Interior, Closure, Boundary, Base, Sub base, Continuity, Topological Equivalence, Subspaces.
(Sections: 4.1, 4.2, 4.3, 4.4 and 4.5)

UNIT IV

Connectedness and disconnected spaces, Theorems on connectedness, Connected subsets of real line, Applications of Connectedness, Path connected spaces.
(Sections: 5.1, 5.2, 5.3, 5.4 and 5.5)

UNIT V

Compact spaces, compactness and continuity, properties related to compactness, one point compactification.
(Sections: 6.1, 6.2, 6.3 and 6.4)

References

1. Gerald Buskes, Arnoud van Rooiji, *Topological Spaces from Distance to Neighbourhood*
2. James R. Munkres, *Topology*, PHI Learning Private Limited, Second Edition, 2009
3. Stephen Willard, *General Topology*, Addison-Wesley, Reading, 1970
4. G.F Simmons, *Topology and Modern Analysis*, Mc Graw-Hill Inc, New York, 13th reprint, 2010.
5. J.Arthur Seebach, Lynn Arthur Steen, *Counter Examples in Topology*, Dover Publications, 1995
6. Sheldon W Davis *Topology*, Tata Mc Graw-Hill Edition 2006, Tata MC Graw-Hill

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APMM221ABSTRACT ALGEBRA

Text: Joseph A Gallian, Contemporary Abstract algebra, 8th Edition, Brooks/Cole, Cengage Learning

Students are introduced to some basic ideas of abstract algebra during their previous courses. In this course, advanced topics of abstract algebra are discussed.

UNIT 1

Groups, Subgroups, Cyclic groups, Permutation groups, Isomorphisms, Cayley’s Theorem, Properties, Automorphisms, Cosets and Lagrange’s Theorem and its consequences.(Chapters 1-7. Proof of theorems in Chapters 1-7 may be omitted). External direct products-Definition and examples, Properties, Groups of units modulo n as an external direct product, Applications, Normal subgroups, Factor groups – Applications, Internal direct product. (Chapters: 8& 9)

UNIT II

Group homomorphisms: Definition, Examples, Properties. First isomorphism Theorem (Chapter 10. Proof of theorems in Chapter 10 may be omitted). Fundamental Theorem of abelian groups, Isomorphism classes of abelian groups, Proof of fundamental theorem (Chapter 11). Sylow theorems, Conjugacy classes, Class Equation, Sylow theorems and applications, Simple groups, examples, non-simplicity tests (Chapters 24 and 25). Theorems 25.1, 25.2, 25.3 and Corollary 1 (Index theorem), Corollary 2 (Embedding theorem) may be discussed without proof.

UNIT III

Rings, Fields, Integral Domain, Characteristics of a ring, Homomorphism –Definition and examples (Chapter 12 and 13. Proof of theorems in Chapter 12 and 13 may be omitted). Ideals, Factor rings, Prime ideals, Maximal ideals, Construction of field of quotients. Factorization of polynomials, Reducibility tests, Irreducibility tests. Unique Factorization in $Z[x]$ (Chapters 14, 15, 17. Irreducibility tests may be discussed without proof. Proof of theorem 17.6 may be omitted).

UNIT IV

Divisibility in Integral domains, Irreducibles, Primes, Historical discussion of Fermat’s last theorem, Unique Factorization Domains, Euclidean domains, Extension fields, Fundamental theorem of field theory, Splitting fields, Zeros of Irreducible polynomials (Chapters 18 and 20).

UNIT V

Algebraic Extensions, Characterization of extensions, Finite extensions, Properties of algebraic extensions, An introduction to Galois Theory- Fundamental theorem of Galois Theory (without proof), Solvability of polynomials by radicals, Insolvability of Quintic.(Chapters 21 and 32. Proof of theorems 32.4 and 32.5 may be omitted)

References:

1. JB Fraleigh, A first course in abstract algebra, Seventh edition, Pearson Education Inc.
2. T W Hungerford, Algebra, Springer 2005
3. I N Heirstein, Topics in Algebra, John Wiley, Inc
4. M Artin, Algebra, Prentice Hall
5. David S. Dummit, Richard M. Foote, Abstract Algebra, Third Edition, Wiley.

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APMM 222 REAL ANALYSIS-II

Text: G.de.Barra, *Measure Theory and Integration*, New Age International Publishers, New Delhi, 1981.

UNIT I

Lebesgue Outer Measure, Measurable sets, Regularity, Measurable functions, Borel and Lebesgue Measurability
(Chapter 2: Sections 1-5)

UNIT II

Integration of Non-negative functions, The General Integral, Integration of Series, Riemann and Lebesgue Integrals, The Four Derivatives, Lebesgue's Differentiation Theorem, Differentiations and Integration.
(Chapter 3: Sections 1-4 ; Chapter 4: Sections 1, 4 – statements only- and 5)

UNIT III

Abstract Measure Spaces: Measures and Outer Measures, Extension of a measure, Uniqueness of the Extension, Completion of the Measure, Measure spaces, Integration with respect to a Measure
(Chapter 5: Sections 1 - 6)

UNIT IV

The L^p Spaces, Convex Functions, Jensen's Inequality, The Inequalities of Holder and Minkowski, Completeness of $L^p(\mu)$.
(Chapter 6: Sections 1-5)

UNIT V

Convergence in Measure, Signed Measures and the Hahn Decomposition, The Jordan Decomposition, The Radon-Nikodym Theorem, Some Applications of the Radon-Nikodym Theorem.
(Chapter 7: Section 1; Chapter 8: Sections 1-4)

References:

1. H.L.Roydon, *Real Analysis, Third Edition*, Mac-Millan
2. W.Rudin, *Principles of Mathematical Analysis*, Third Edition
3. P.R Halmos, *Measure Theory*, Springer.

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APMM 223 TOPOLOGY-II

- Texts: 1. Principles of Topology by Fred H. Croom, Baba Barkha Nath Printers (India), Third Reprint 2009.**
2. Topology by Sheldon W. Davis, Tata Mc Graw-Hill Edition, 2006

UNIT I

Product and Quotient Spaces: Finite and arbitrary products, Comparison of topologies, Quotient spaces
(Chapter 7: Sections 1-4 of Text-1 (excluding Alexander sub basis theorem and Theorem-7.11))

UNIT II

Separation axioms: T_0 , T_1 and T_2 – spaces, Regular spaces, Normal spaces, Separation by continuous functions.
(Chapter 8: Sections 1-4 of Text-1)

UNIT III

Convergence, Tychonoff’s Theorem.
(Chapter 16: Theorem 18.21 and Theorem 18.22 of Text-2)

UNIT IV

Algebraic Topology: The fundamental group, The fundamental group of S^1 .
(Chapter 9: Sections 1-3 of Text-1)

UNIT V

Examples of fundamental groups, The Brouwer Fixed Point Theorem.
(Chapter 9: Sections 4-5 of Text-1)

References

1. Gerald Buskes, Arnoud van Rooij, Topological Spaces from Distance to Neighbourhood
2. James R. Munkres, Topology, PHI Learning Private Limited, Second Edition, 2009
3. Stephen Willard, *General Topology*, Addison-Wesley, Reading, 1970
4. G.F Simmons, *Topology and Modern Analysis*, Mc Graw-Hill Inc, New York, 13th reprint, 2010.
5. J.Arthur Seebach, Lynn Arthur Steen, *Counter Examples in Topology*, Dover Publications, 1995
6. Sheldon W Davis *Topology*, Tata Mc Graw-Hill Edition 2006, Tata MC Graw-Hill

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APMM 224 SCIENTIFIC PROGRAMMING WITH PYTHON

Texts: 1. Jaan Kiusalaas, *Numerical Methods in Engineering with Python3*, Cambridge University Press.

2. Amit Saha, *Doing Math with Python*, No Starch Press, 2015

UNIT I

This unit is based on sections 1.1 to 1.5 of text-1. Even though some of the materials may be familiar to students, a quick review should be given as in the sections. For a detailed review, it is recommended to consider chapters 4, 5, 6, 8 and 9 of reference text-1. Importance should be given for defining functions and importing functions from modules.

UNIT II

Visualizing Data with Graphs – Learn a powerful way to present numerical data – by drawing graphs with Python. The unit is based on Chapter 2 of Text-2. The sections Creating Graphs with Matplotlib and Plotting with Formulas must be done in full. In the Section, Programming Challenges, the Problems Exploring a Quadratic Function Visually, Visualizing your Expenses and Exploring the Relationship between the Fibonacci Sequence and Golden Ratio must be discussed.

UNIT III

This unit is based on chapters 4 and 7 of Text-2. Here we discuss Algebra and Symbolic Math. with SymPy and Solving Calculus Problems. In chapter-4, the Sections, Defining Symbols and Symbolic Operations, Working with Expressions, Solving Equations and Plotting using SymPy should be done in full. In the Section, Programming Challenges, the problems Factor Finder, Graphical Equation Solver, summing a Series and Solving Single-Variable Inequalities also should be discussed.

In chapter-7, some problems discussed, namely, Finding the Limit of Functions, Finding the Derivative of Functions, Higher-Order Derivatives and Finding the Maxima and Minima and Finding the Integrals of Functions are to be done. In the section, Programming Challenges, the problems Verify the Continuity of a Function at a Point, Area Between Two Curves and Finding the Length of a Curve also should be discussed.

UNIT IV

Gauss Elimination Method (excluding Multiple Sets of Equations), LU Decomposition Methods (Doolittle's Decomposition Method only)

[Sections 2.2, 2.3 of Text-1]

Interpolation and Curve Fitting – Polynomial Interpolation – Lagrange’s Method, Newton’s divided difference method and Limitations of Polynomial Interpolation.

(Sections: 3.1 and 3.2 of Text-1)

Roots of Equations – Method of Bisection and Newton-Raphson Method

(Sections: 4.1, 4.3 and 4.5 of Text-1)

UNIT V

Numerical Integration – Newton-Cotes Formulas - Trapezoidal Rule, Simpson’s Rule and Simpson’s 3/8 Rule

(Sections: 6.1, 6.2 of Text-1)

Initial Value Problems – Euler method and Runge-Kutta Methods

(Sections: 7.1, 7.2, 7.3 of Text-1)

For more problems visit: <https://www.nostarch.com/doingmathwithpython/>,
<https://doingmathwithpython.github.io/author/amit-saha.html> and <https://projecteuler.net/>.

1. The course is aimed to give an introduction to mathematical computing, with python as tool for computation.
2. The students should be encouraged to write programs to solve the problems given in the sections as well as in the exercises.
3. The end semester evaluation should contain a theory and a practical examination.
4. The duration of the theory examination will be three hours, with a maximum of 50 marks. One question out of two from each unit has to be answered. Each carries 10 marks.
5. In the question paper for theory examination, importance should be given to the definition, concepts and method discussed in each unit, and not for writing long programs.
6. Practical examination shall also be of two hours duration for a maximum of 25 marks with 5 questions carrying equal marks.
7. Continuous evaluation follows the pattern – 5 marks for attendance, 10 marks for the internal examination (theory) and 10 marks for the practical record. The record should contain at least 20 programs.
8. Practice of writing the record should be maintained by each student throughout the course and it should be dually certified by the teacher- in charge/internal examiner and evaluated by the external examiner of practical examination.

Reference

1. Vernon L. Ceder, The Quick Python Book, Second Edition, Manning.
2. **NumPy Reference Release 1.12.0**, Written by the NumPy community. (Available for free download at <https://docs.scipy.org/doc/numpy-dev/numpy-ref.pdf>).
3. S. D, Conte and Carl de Boor, **ELEMENTARY NUMERICAL ANALYSIS An Algorithmic Approach**, Third Edition, McGraw-Hill Book Company.

4. S. S. Sastry, *Introductory Methods of Numerical Analysis*, Fifth Edition, PHI.

APMM 321 COMPLEX ANALYSIS- I

Text: John. B. Conway, *Functions of Complex Variables, Springer- Verlag, New York,1973.*

Unit I

Elementary properties and examples of analytic functions, Power series, Analytic functions, Riemann Stieltjes Integrals.

(Chapter3: Sections 1,2; Chapter 4: Section 1)

Unit II

Power series representation of an analytic function, zeros of an analytic function,The index of a closed curve.

(Chapter 4: Sections 2,3 and 4)

Unit III

Cauchy’s theorem and integral formula, Homotopic version of Cauchy’s theorem, Simple connectivity, Counting zeros, The open mapping theorem, Goursat’s Theorem.

(Chapter 4: Sections 5, 6, 7 and 8 (avoid the proof of theorem 6.7))

Unit IV

Classification of singularities, Residues (Example 2.12 only for review), The augment Principle.

(Chapter 5: Sections 1, 2 and 3)

Unit V

The extended plane and its spherical representation, Mobius transformations, The Maximum Principle, Schwarz’s Lemma.

(Chapter 1: Section 6; Chapter 3: Section 3; Chapter 6: Sections 1 and 2)

References:

1. L.V. Ahlfors, *Complex Analysis*, Mc-Graw Hill (1966)
2. S. Lang, *Complex Analysis*, Mc-Graw Hill (1998).
3. S. Ponnusamy & H.Silverman, *Complex Variables with Applications*, Birkhauser
4. H.A. Priestley, *Introduction to Complex Analysis*, Oxford University Press Tristan Needham, *Visual Complex Analysis*, Oxford University Press (1999)
5. V. Karunakaran, *Complex Analysis*, Narosa Publishing House

APMM 322 FUNCTIONAL ANALYSIS – I

Text: B.V Limaye, *Functional Analysis* (3rd Edition)

UNIT I

Normed Spaces and Continuity of Linear maps.
(Sections 5 and 6 of the Text, omitting 6.5(d) and 6.8)

UNIT II

Hahn-Banach Theorem and Banach Spaces.
(Sections 7 and 8 of the Text, Theorem-7.12 statement only)

UNIT III

Uniform Bounded Principle, Closed and Open Mapping Theorems
(Sections 9.1, 9.2, 9.3 and 10 of the Text)

UNIT IV

Bounded Inverse Theorem, Spectrum of a Bounded Operator
(Sections 11.1, 11.3, 12 (excluding 12.4) and 13.1 of the Text)

UNIT V

Weak Convergence, Reflexivity, Compact Linear Maps (Sections 15.1, 15.2(a), 16.1, 16.2, 17.1, 17.2, 17.3 and 17.4(a) of the Text)

References

1. Bryan Rynne, M A Youngson, *Linear Functional Analysis*, Publisher: Springer
2. Rajendra Bhatia, *Notes on Functional Analysis*, Publisher: Hindustan Book Agency
3. M Thamban Nair, *Functional Analysis: A First Course*, Publisher: Prentice Hall of India Pvt. Ltd.
4. Walter Rudin, *Functional Analysis*, 2nd Edition, Publisher: Tata Mc Graw Hill
5. B. V. Limaye, *Linear Functional Analysis for Scientists and Engineers*, Springer Singapore, 2016.

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APMM 323 AUTOMATA THEORY (Elective-1)

Text: J.E. Hopcroft and J.D. Alan, *Introduction to Automata Theory Languages and Computation*, Narosa, 1999.

UNIT I

Strings, Alphabets and Languages (Section 1.1 of the Text)

Finite Automata (Chapters 2, Sections 2.1 to 2.4)

UNIT II

Regular expressions and Properties of Regular sets.

(Sections 2.5 to 2.8 and 3.1 to 3.4)

UNIT III

Context Free grammars (Section 4.1 to 4.5)

UNIT IV

Pushdown Automata & properties of Context free languages

Theorem 5.3, 5.4 (without proof), (Section is 5.1 to 5.3 and 6.1 to 6.3)

UNIT V

Turning Machine and Choamski hierarchy, (Sections 7.1 to 7.3 and 9.2 to 9.4)

References

1. G.E Revesz , *Introduction to Formal Languages*
2. P.Linz , *Introduction to Formal Languages and Automata*, Narosa 2000
3. G.Lallment, *Semigroups and Applications*

APMM 323 APPROXIMATION THEORY (Elective-1)

Text: EW Cheney, *“Introduction to Approximation Theory”*, Mc Graw Hill

UNIT 1

Metric spaces- An existence Theorem for best approximation from a compact subset; Convexity- Caratheodory's Theorem- Theorem on linear inequalities; Normed linear spaces - An Existence Theorem for best approximation from finite dimensional subspaces - Uniform convexity - Strict convexity.

(Sections 1,2,5,6 of Chapter 1)

UNIT 2

The Tchebycheff solution of inconsistent linear equations - Systems of equations with one unknown-Three algebraic algorithms; Characterization of best approximate solution for m equations in n unknowns- The special case $m = n + 1$; Poly's algorithm.

(Section 1,2,3,4,5 of Chapter 2)

UNIT 3

Interpolation- The Lagrange formula-Vandermonde's matrix- The error formula- Hermite interpolation;The Weierstrass Theorem- Bernstein polynomials- Monotone operators- Fejer's Theorem; General linear families- Characterization Theorem- Haar conditions- Alternation Theorem.

(Sections 1,2,3,4, of Chapter 3)

UNIT 4

Rational approximation- Conversion of rational functions to continued fractions; Existence of best rational approximation- Extension of the classical Theorem; Generalized rational approximation- The characterization of best approximation- An alternation Theorem- The special case of ordinary rational functions; Unicity of generalized rational approximation.

(Sections 1,2,3,4 of Chapter 5)

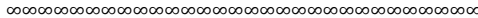
UNIT 5

The Stone Approximation Theorem, The Muntz Theorem - Gram's lemma, Approximation in the mean-Jackson's Unicity Theorem- Characterization Theorem, Marksoff's Theorem.

(Section 1,2,6 of Chapter 6)

Reference:

P.J Davis. *“Interpolation and Approximation”*, Blaisdell Publications.



APMM 323 OPERATIONS RESEARCH (Elective-1)

- Texts:** 1) Ravindran, Philips, Solberg, *Operations Research, Principles and Practice*, Second Edition, John Wiley & Sons.
2) K. V. Mital, C. Mohan. *Optimization Methods in Operations Research and Systems Analysis*, Third Edition, New Age International Publishers, New Delhi.

UNIT I

Linear Programming: Formulation of Linear Programming Models, Graphical solution of Linear Programs in two variables, Linear programs in standard form, basic variable, basic solution, basic feasible solution, Solution of Linear Programming problem using simplex method, Big – M simplex method, The two-phase simplex method.
[Chapter 2 of text 1, sections 2.1 to 2.9]

UNIT II

Transportation Problems: Linear programming formulation, Initial basic feasible solution, degeneracy in basic feasible solution, Modified distribution method, Optimality test.
Assignment Problems:
Standard assignment problems, Hungarian method for solving an assignment problem.
[Chapter 3 of text 1, sections 3.1 to 3.3]

UNIT III

Project management; Programme Evaluation and Review Technique (PERT), Critical Path Method (CPM)
[Chapter 3 of text 1, section 3.7]

UNIT IV

Kuhn – Tucker Theory and Non-linear Programming: Lagrangian function, saddle point, Kuhn–Tucker conditions, Primal and dual problems, Quadratic Programming.
[Chapter 8 of text 2, sections 1 to 6]

UNIT V

Dynamic Programming: Minimum path, Dynamic Programming problems, Computational economy in DP, serial multistage model, Examples of failure, Decomposition, Backward recursion.
[Chapter 10 of text 2, sections 1 to 10]

Reference:

1. Hamdy A. Taha, *Operations Research*, Fifth edition, PHI
2. SR Yadav and AK Malik, *Operations Research*, Oxford University Press, 2014
3. Kanti Swarup, P. K. Gupta, Man Mohan, *Operations Research*, Sultan Chand & Sons, 1978

4. Sharma J K, Operations Research: Theory and Applications 5th Edition, Macmillan India Limited, 2013

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APMM 323 MATHEMATICAL STATISTICS (Elective-1)

Texts:

- (1) Lehmann.E.L *“Theory of Point Estimation”*, John wiley, New York (1983).
- (2) Lehmann.E.L *“Testing of Statistical Hypothesis”*, John Wiley, New York (Second Edition 1986)
- (3) Randles R.H and Wolf D.A-*“Introduction to the Theory of Non Parametric Statistics”*, Wiley, New York (1979)
- (4) Kendall, M.G and Stuart. A, *“The Advanced Theory of Statistics”*, Vol. 2 Mac Millan , New York (Fourth Edition 1979)

UNIT I

Problem of point estimation, General properties of estimators unbiasedness, Strong weak and squarederror consistency, A sufficient condition for weak consistency, UMVU estimators, BLUE’s, Sufficiencyand completeness, Exponential family of densities and complete sufficient statistic, Statement of Fisher-Neyman factorization Theorem (Without proof)

UNIT II

Rao-Blackwell Theorem, Lehmann-Scheffe Theorem and their application to derive UMVU estimators,Ancillary statistic and Basu;s Theorem, Cramer Rao inequality

UNIT III

Least square estimators, Maximum likelihood estimators and estimators by the method of moments andtheir properties.

UNIT IV

Tests of hypothesis: Null and alternate hypotheses, Two kinds of errors, Level of significance Power oftest, Power function, Size of a test, Test of a simple hypothesis against a simple alternate hypothesis,Leyman-Pearson Lemma, Test of a composite hypothesis against Composite alternate hypothesis,Likelihood Ratio Test.

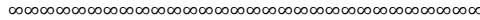
UNIT V

Non- Parametric Methods Chi Square Test of goodness of fit, Empirical distribution function, $F_n(x)$ as an estimator of population distribution function $F(x)$, its exact and asymptotic distributions for fixed x ,Koimogrove test, Sign test, Wilcoxon – Mann-Whitney Test

References:

1. Rohatgi V.K , *“ An Introduction to Probability Theory and Mathematical Statistics”*, Wiley Eastern New Delhi (1985)
2. Rao, C.L , *“ Liner Statistical Inference and Its Applications ”*, Wiley Eastern , New Delhi (1974)

- Mood Ali, Gray Bill Fhardoes D.C” , “*Introduction to the Theory of Statistics*”, Mc Graw Hill International, New York (Third Edition, 1972)



APMM 324: FIELD THEORY (Elective-2)

Text: Joseph Rotman, *Galois Theory*, Second Edition, Springer, 1998.

UNIT 1

Solvable groups (Appendix B of the text): Isomorphism Theorems, Correspondence Theorem, Sylowp-subgroup, commutator subgroups and Higher subgroups, S_5 is not solvable.

(The following results are included: G5 , G6 , G7 , G8 , G9 , G14 , G15 , G16 , G17 , G18 , G19 , G20 , G21 , G22,G23 , G31 , G34 , G36 , G37 , G38 ,G39)

UNIT 2

Polynomial Rings over Fields: Principal ideal, Greatest common divisor, LCM, Remainder Theorem,Prime and maximal ideals, Splitting, prime fields, Characteristic, Irreducible and primitive polynomials,Content, Eisenstein Criterion, Cyclotomic polynomial.

(The following results are included: Theorem 13 to Theorem 22, Theorem 24 to Theorem 33, Theorem 35 to Corollary 42)

UNIT 3

Splitting Fields: Degree of an extension, Simple extension, Algebraic extension and transcendental extension,Splitting field, Separable extension, Galois field, Galois group.

(The following results are included: Lemma 44 to Corollary 53, Lemma 54 to Theorem 58)

UNIT 4

Roots of Unity and Solvability by Radicals: Cyclic group of nth roots of unity, Primitive element,Fobenius automorphism, Radical extension, Solvability by radicals, Unsolvable quintic.

(The following results are included: Theorem 62 to Corollary 72, Lemma 73 to Theorem 75)

UNIT 5

Fundamental Theorem of Galois Theory: Galois extensions, Fundamental Theorem, FundamentalTheorem of Algebra, Galois Theorem on solvability.

(The following results are included: Theorem 81 to Corollary 93, Lemma 94 to Theorem 98)

References:

- Harold M. Edwards, *Galois Theory*, Springer, 1984.
- Joseph . A. Gallian, *Contemporary Abstract Algebra*, 7th Edition, Brooks/Cole.

3. J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Edition, PHI.
4. T. W. Hungerford, *Algebra*, Springer 2005.
5. O. Zariski and P. Samuel, *Commutative Algebra*, Vol. I, Springer – Verlag

oo

APMM 324: GRAPH THEORY (Elective-2)

Text: Gary Chartrand and Ping Zhang , *Introduction to Graph Theory*, Tata Mc Graw Hill, Edition 2006

An overview of the concepts-Graphs, Connected graphs, Multi graphs, Degree of a vertex, Degree Sequence, Trees.

UNIT I

Definition of isomorphism, Isomorphism as a relation, Graphs and groups, Cut-vertices, Blocks, Connectivity.
(Sections 3.1, 3.2, 3.3, 5.1, 5.2 and 5.3)

UNIT II

Eulerian graphs, Hamilton graphs, Hamilton walks and numbers
(Sections 6.1, 6.2 and 6.3)

UNIT III

Strong diagraphs, Tournaments, matching, Factorization.
(Sections 7.1, 7.2, 8.1 and 8.2)

UNIT IV

The Four colour problem, Vertex colouring, The Ramsey number of graphs, Turan’s Theorem,
(Sections 10.1, 10.2, 10.3, 11.1 and 11.2)

UNIT V

The centre of a graph, Distant vertices, Locating numbers, Detour and directed distance.
(Sections 12.1, 12.2, 12.3, 12.4, 12.5 and 12.6)

References:

1. Bondy J.A and Murthy U.S.R, “*Graph Theory with Applications*”, the Macmillan Press Limited.

2. Hararay F., "*Graph Theory*", Addison-Wesley
 3. Suesh Singh G., "*Graph Theory*", PHI Learning Private Limited
 4. Vasudev.C , "*Graph Theory Applications*".
 5. West D.B,"*Introduction to Graph Theory*", PHI Learning Private Limited
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APMM 324: MECHANICS (Elective-2)

Text: Herbert Gioldstein, *Classical Mechanics*, Addison Wesley

UNIT I

Mechanics of a particle, Mechanics of a system of particles, Constraints, D'Alembert's principles and Lagange's Equations, Velocity dependent potentials and dissipation functions, Simple applications of Lagrangian formulation.

(Chapter 1 of Text)

UNIT II

Hamilton's principle, Derivation of Lagrange's equation, Some techniques of Calculus of Variation, Extension of Hamilton principle, Conservation Theorems.

(Sections: 2.1, 2.2, 2.3, 2.4 and 2.6 of Text)

UNIT III

The two body Central force problem, Reduction to equivalent one body problem equation of notation, The equivalent one dimensional problem, The Virial Theorems, the differential equations for the orbits, The Keplar problem.

(sections 3.1 to 3.6 of Text)

UNIT IV

The Kinematics of rigid body motion, the independent coordinates of a rigid body orthogonal transformations, The Eulerian angles, The Cayley-Klein parameters, Euler's Theorem on the motion of a rigid body, The Coriolis force

(Sections 4.1, 4.2, 4.4, 4.5, 4.6, 4.9 of Text)

UNIT V

The rigid body equations of motion, Angular momentum, Tensor and dynamics, The inertia tensor, The eigen values of the inertia tensor, Methods of solving rigid body problem and Euler equations of motion.
(Sections 5.1 to 5.6 of Text)

Reference:

Synge J.L and Griffith B.A, *Principles of Mechanics*, MC Graw-Hill

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APMM 324: THEORY OF WAVELETS (Elective-2)

Text Book:Michael Frazier, *An Introduction to Wavelets through Linear Algebra*, Springer
Prerequisites: Linear Algebra, Discrete Fourier Transforms, elementary Hilbert Space Theorems
(No questions from the pre-requisites)

UNIT I

Construction of Wavelets on Z_n the first stage. (Section 3.1)

UNIT II

Construction of Wavelets on Z_n the iteration sets, Examples - Shamon, Daubichie and Haar
(Sections: 3.2 and 3.3)

UNIT III

$l_2(Z)$, Complete Orthonormal sets, $L_2[-\pi,\pi]$ and Fourier Series.
(Sections: 4.1,4.2 and 4.3)

UNIT IV

Fourier Transforms and convolution on $l_2(Z)$, First stage wavelets on Z .
(Section: 4.4 and 4.5)

UNIT V

The iteration step for wavelets on Z , Examples, Shamon Haar and Daubiehie

References:

Mayor (1993), *Wavelets and Operators*, Cambridge University Press

Chui. C(1992), *An Introduction to Wavelets*, Academic Press, Boston

oo

APMM 421 COMPLEX ANALYSIS -II

Text: John. B. Conway, *Functions of Complex Variables*, Springer- Verlag, New York,1973.

Unit I

Compactness and convergence in the space of analytic functions. The space $C(G,\Omega)$, Space of analytic functions,Riemann Mapping Theorem(Lemma 4.3, statement only).

(Chapter 7: Sections 1, 2 and 4)

Unit II

Weierstrass factorization Theorem, Factorization of sine function, The Gamma function (Only statements of theorem 7.15 and lemmas 7.16, 7.17, and 7.19)

(Chapter 7: Section 5, 6 and 7)

Unit III

Riemann Zeta function, Runge's Theorem, Simple Connectedness, Mittag Leffler's Theorem.

(Chapter 7: Section 8 (excluding 8.13 and 8.14) and Chapter 8)

Unit IV

Schwarz Reflection Principle, Analytic continuation along a path, Monodromy theorem.

(Chapter 9: Sections 1, 2 and 3)

Unit V

Basic properties of harmonic functions, Harmonic functions on a disc, Jensen's formula, The genus and order of an entire function, Hadamard factorization Theorem.

(Chapter 10: Sections 1(Maximum Principle- second version - statement only), 2;

Chapter 11: Sections 1, 2 (Theorem 2.6 statement only), and 3 (Lemma 3.1 statement only).

References

1. L.V. Ahlfors, *Complex Analysis*, Mc-Graw Hill (1966)
2. S. Lang, *Complex Analysis*, Mc-Graw Hill (1998).
3. S. Ponnusamy & H.Silverman, *Complex Variables with Applications*, Birkhauser
4. H.A. Priestley, *Introduction to Complex Analysis*, Oxford University Press Tristan Needham, *Visual Complex Analysis*, Oxford University Press (1999)
5. V. Karunakaran, *Complex Analysis*, Narosa Publishing House

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APMM 422 FUNCTIONAL ANALYSIS– II

Texts: 1) B.V Limaye, *Functional Analysis (3rdEdition)*

UNIT I

Spectrum of a compact operator
(Sections 18.1-18.5 and 18.7(a))

UNIT II

Inner Product Spaces, Orthonormal Sets. (Section 21 and 22, omitting 21.3(d), 22.3(b), 22.8(b), 22.8(c), 22.8(d), 22.8(e))

UNIT III

Approximation and Optimization, Projection and Riesz Representation Theorems
(Section 23 and 24, omitting 23.6)

UNIT IV

Bounded Operators and Adjoints - Normal, Unitary and Self-Adjoint Operators
(Section 25 and 26.1, 26.5 omitting 25.4(b))

UNIT V

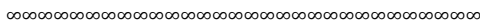
Spectrum and Numerical Range - Compact Self-Adjoint Operators
(Section 27.1, 27.5, 27.7, 28.1, 28.4, 28.5, 28.6)

References:

1. Bryan Rynne, M A Youngson, *Linear Functional Analysis*, Publisher: Springer
2. Rajendra Bhatia, *Notes on Functional Analysis*, Publisher: Hindustan Book Agency

References

1. E.R Berlekamp, *Algebraic Coding Theory*, Mc Graw-Hill, 1968
2. P.J Cameron and J.H Van Lint, *Graphs, Codes and Designs* CUP
3. H. Hill, *A First Course in Coding Theory*, OUP 1986



APMM 423 DIFFERENTIAL GEOMETRY (Elective-3)

Text: John.A. Thorpe, *Elementary Topics in Differential Geometry*, Springer Verlag

UNIT I

Graphs and level sets, Vector fields, Tangent Spaces . (Chapter 1,2,3 of Text)

UNIT II

Surfaces, Vector fields on surfaces, Orientation, The Gauss map (Chapter 4,5 6 of Text)

UNIT III

Geodesics, Parallel transport (Chapter 7,8 Text)

UNIT IV

The Weingarten map, Curvature of plane curve. (Chapter 9.1 of Text)

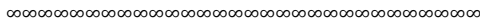
UNIT V

Arc length, Line integral, Curvature of surfaces (Chapter 11,12 of Text)

References:

1. I Singer and J.A Thorpe, *Lecture notes on Elementary Topology and Geometry*, Springer-Verlag

2. M Spivak, *Comprehensive introduction to Differential Geometry (Vols 1 to 5)*, Publish or Perish Boston.



APMM 423 GEOMETRY OF NUMBERS (Elective-3)

Text Book: D.D Olds, Anneli Lax and Guiliana P. Davidoff, *The Geometry of Numbers*, The Mathematical Association of America 2000

UNIT 1

Lattice points and straight lines, Counting of lattice points (Chapters 1 and 2)

UNIT 2

Lattice points and area of polygons, Lattice points in circles (Chapter 3 and 4)

UNIT 3

Minkowski Fundamental Theorem and Applications (Chapters 5 and 6)

UNIT 4

Linear transformation and integral lattices, Geometric interpretations of Quadratic forms (Chapters 7 and 8)

UNIT 5

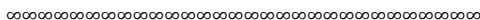
Blichfeldts and applications, Tchebychev's and consequences (Chapter 9 and 10)

References

1. J.W.S Cassells, *Introduction to Geometry of Numbers*, Springer Verlag 1997

References:

1. Bondy and Murthy, *Graph Theory with Applications*, The Macmillan Press Limited, 1976
2. Chartrand G and L.Lesniak, *Graphs and Diagraphs*, Prindle, Weber and Schmidt, Boston1986
3. Garey M.R, D.S Johnson , *Computers and Intractability*, A Guide to the Theory of NP-Completeness, Freeman, San Francisco 1979.
4. Harary. F, *Graph Theory*, Addison Wesley Reading Mass 1969 (Indian Edition, Narosa)
5. K.R Parthasarathy, *Basic Graph Theory*, Tata Mc Graw-Hill, Publishing Co,New Delhi, 1994.



APMM 424 ANALYTIC NUMBER THEORY (Elective-4)

Text: Tom M. Apostol; *Introduction to Analytical Number Theory*, Springer-Verlag

UNIT I

The Fundamental Theorem of Arithmetic (chapter 1)

UNIT II

Arithmetical function and Dirichlet multiplication(Section 2.1 to 2.17)

UNIT III

Congruences, Chinese Remainder Theorem(Sections 5.1 to 5.10)

UNIT IV

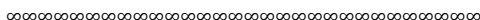
Quadratic residues, Reciprocity law, Jacobi symbol (Sections 9.1 to 9.8)

UNIT V

Primitive roots, Existence and number of primitive roots, Prime Number Theorem. (Sections 10.1 to 10.9)

References

1. Emd Groswald, *Topics from the Theory of Numbers*, Birkhause
2. G.H Hardy and E.M Wright, *Introduction to the Theory of Numbers*, Oxford



APMM 424 CATEGORY THEORY (Elective-4)

Text Book: S. MacLane, *Categories for the working Mathematician*, Springer, 1971

UNIT-I

Categories, Functors and Natural Transformations - Axioms for categories, categories, Functors. Natural Transformations, Monoids, Epimorphisms and Zero Foundations, Large Categories, Hom-sets.

UNIT II

Constructions on categories - Duality Contravariance and opposites, Products of Categories. Functor Categories, The category of all categories, Comma categories, Graphs and Free categories, Quotient Categories.

UNIT III

Universals and Limits - Universal Arrows, Yoneda Lemma Coproducts and Colimits, Products and Limits, Categories with Finite products, Groups in categories.

UNIT IV

Adjoints – Adjunctions, Examples of Adjoints, Reflective subcategories, Equivalence of categories, Adjoints for pre orders, Cartesian closed categories, Transformations of Adjoints, Compositions of Adjoints.

UNIT –V

Limits – Creation of Limits by products and Equalizers, Limits with parameters, Preservation of Limits, Adjoints on Limits, Freyd's Adjoint Functor Theorem, Subobjects and Generation, The Special Adjoint Functor Theorem, Adjoint in Topology.

Reference:

1. M.A. Arbib and E.G Maneswarrows, *Structures and Functors*, The categorical Imperative,Avademic Press-1975
2. H. Herrlich and G.E Strecker, *Category Theory*, Allyn & Bacon, 1973
3. M. Barmand, C. Wells, *Category Theory for Computer Science*, Prentice Hall , 1990
4. F. Borceux, *Handbook of Categorical Algebra*, Vol. I, II, III, Cambridge, University Press,1994
5. P. Frevd, *Abelian Categories*, Harper & Row, 1964
6. R.F,C Walters, *Categories and Computer Science*, Cambridge University Press, 1991

oo

APMM 424 COMMUTATIVE ALGEBRA (Elective-4)

Text: N.S Gopalakrishan, *Commutative Algebra*, Oxonian Press

UNIT I

Modules, Free projective, Tenser product of modules, Flat modules

(Chapter 1 of Text)

UNIT II

Ideals, Local rings, Localization and applications

(Chapter 2 of Text)

UNIT III

Noetherian rings, modules, Primary decomposition, Artinian modules

(Chapter 3 of Text)

UNIT IV

Integral domains, Integral extensions, Integrally closed domain, Finiteness of integral closure

(Chapter 4 of Text)

UNIT V

Valuation rings, Dedikind domain

(Chapter 5 of Text, Theorems 4 and 5 omitted)

Reference:

1. M.F Atiyah and I.G Mac Donald, *Introduction to Communication Algebra*, Addison Wesley

2. T.W Hungerford, *Algebra*, Springer-Verlag

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APMM 424 REPRESENTATION THEORY OF FINITE GROUPS(Elective-4)

Text: Walter Ledermann, *Introduction to Group Characters*, Cambridge University Press

UNIT I

G-module, Characters, Reducibility, Permutation representations, Complete reducibility, Schur's Lemma

(Sections 1.1 to 1.7 of Text)

UNIT II

The commutant algebra, Orthogonality relations, The groups algebra

(Section 1.8, 2.1, 2.2 of Text)

UNIT III

Character table, Character of finite abelian groups, The lifting process, Linear characters.

(Section 2.3, 2.4, 2.5, 2.6 of Text)

UNIT IV

Induced representations, Reciprocity law, A_5 , Normal subgroups, Transitive groups, Induced characters of S_n

(Sections 3.1, 3.2, 3.3, 3.4, 4.1, 4.2, 4.3 of Text)

UNIT V

Group theoretical applications, Burnside's (p,q) Theorem, Frobenius groups.

(Chapter 5 of Text)

Reference:

S.Lang, *Algebra*, Addison Wesley

oo