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MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :…………………….**

Third Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Core Course: Mathematics – II

AUMM341: Algebra and Calculus I

(*for 2014 Admissions – **Improvement** Only*)

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer **ALL** questions / problems in one or two sentences.*

1. Give an example for a commutative ring with unity.
2. Write all zero divisors in $\mathbb{Z}/10\mathbb{Z}$.
3. Find the order of [2] in $\mathbb{Z}/7\mathbb{Z}$.
4. Show that a group G is abelian if $(a * b)^2 = a^2 * b^2$ for every a, b $\in$ G.
5. Find the remainder obtained when 8^{103} is divided by 13.
6. Find the acute angle between the vectors $\bar{u} = 2\bar{i} + 3\bar{j} - 6\bar{k}$ and $\bar{v} = 2\bar{i} + 3\bar{j} + 6\bar{k}$.
7. Find the equation of the paraboloid $z = x^2 + y^2$ in cylindrical co – ordinates.
8. Find the parametric equations for the line segment joining the points P_1 (2, 4, –1) and P_2 (5, 0, 7).
9. Describe the surface: $4x^2 + 4y^2 + z^2 + 8y - 4z = -4$.
10. Find the spherical co – ordinates of the point that has rectangular co – ordinates $(4, -4, 4\sqrt{6})$.

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions / problems, not exceeding a paragraph.*

11. If $\mathbb{Z}/m\mathbb{Z}$ is a field, prove that m is prime.

12. Prove that every subgroup of an abelian group is normal.
13. Prove that a field has no zero divisors.
14. Find the inverses of [53] and [113] in $\mathbb{Z}/365\,\mathbb{Z}$.
15. Show that if p is prime, then $(p - 1)!$ is relatively prime to p.
16. Find all solutions of $x^2 \equiv 1 \pmod{35}$.
17. Show that the direction cosines of a vector satisfy $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.
18. Find two unit vectors that are orthogonal to both $\bar{u} = -7\bar{i} + 3\bar{j} + \bar{k}$ and $\bar{v} = 2\bar{i} + 4\bar{k}$.
19. Express the equation $x^2 + y^2 + z^2 = 2z$ in spherical co – ordinates.
20. Find the equation of the plane passing through the origin and parallel to the plane $4x - 2y + 7z + 12 = 0$
21. The planes $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$ are parallel. Find the distance between them.
22. Find the volume of the parallelopiped with adjacent edges $\bar{u} = 3\bar{i} + 2\bar{j} + \bar{k}$ $\bar{v} = \bar{i} + \bar{j} + 2\bar{k}$ and $\bar{w} = \bar{i} + 3\bar{j} + 3\bar{k}$.

(8 × 2 = 16 Marks)

SECTION – C

Short essay type problems : Answer any ***SIX*** *questions.*

23. In $\mathbb{Z}/26\mathbb{Z}$, find the inverses of [9], [11] and [19].
24. Show that if R is a ring and a is any element of R, then a.0 = 0.
25. Show that f: R $\rightarrow$ S is a homomorphism and if a is a unit of R, then $f(a)$ is a unit of S.
26. Show that for any n, $n^{111} \equiv n \pmod{11}$.
27. Find the equation in rectangular co – ordinates and sketch the graph of $\rho \sin\varphi = 1$.
28. Prove the vector identity $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = ((\bar{a} \times \bar{b}).\,\bar{d})\bar{c} - ((\bar{a} \times \bar{b}).\,\bar{c})\bar{d}$.
29. Show that the line $x = 0$, $y = t$, $z = t$ lies in the plane $6x + 4y - 4z = 0$ and parallel to $5x - 3y + 3z = 1$.
30. Find the equation of the plane passing through the point (3, –1, 7) and perpendicular to the vector $\bar{n} = (4, 2, -5)$.
31. Describe the region $0 \le \varphi \le \dfrac{\pi}{6}, 0 \le \rho \le 2$ in the 3D space.

(6 × 4 = 24 Marks)

SECTION – D

*Long essay type problems : Answer any **TWO** questions.*

32. Show that the set Q [i] = {a + ib; a, b ∈ Q} is a field.

33. Find the order of every non – zero element of $\mathbb{Z}/19\mathbb{Z}$.

34. i). Show that the curve of intersection of the surfaces $z = \sin\theta$ and $r = a$ is an ellipse.

    ii). Sketch the surface $z = \sin\theta$; for $0 \leq \theta \leq \frac{\pi}{2}$.

35. Find the vector with length 5 and direction angles $\alpha = 60°$, $\beta = 120°$, $\gamma = 135°$.

(2 × 15 = 30 Marks)

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