



MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :.....

Name :.....

Third Semester B.Sc. Degree Examination, November 2015

First Degree Programme under CBCSS

Core Course: Mathematics – II

AUMM341: Algebra and Calculus I

Time: 3 Hours

Max. Marks: 80

SECTION – A

Answer ALL questions / problems in one or two sentences.

1. True or False: In \mathbb{Q} every non – zero rational is a unit.
2. Find the order of [2] in $\mathbb{Z} / 5\mathbb{Z}$.
3. Define a ring homomorphism.
4. Find the inverse of 3 in $\mathbb{Z} / 5\mathbb{Z}$.
5. State Chinese Remainder Theorem.
6. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y + 8z + 17 = 0$.
7. Find the norm of the vector $v = (1, -1)$.
8. Write the parametric equation of a line passing through $(1, 2, -3)$ and parallel to $2i - 3j + k$.
9. Convert the rectangular coordinates $(1, \sqrt{3}, -2)$ to spherical coordinates.
10. Identify the surface: $z^2 - x^2 - \frac{y^2}{4} = 1$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any EIGHT questions / problems, not exceeding a paragraph.

11. Let M be the ring of 2×2 matrices with real entries under the addition and multiplication of matrices. Show by examples that M is not commutative and that M has zero divisors.

P.T.O.

12. Define a subgroup of a group. Is the union of two subgroups always a subgroup ? Justify.
13. Prove that a ring homomorphism f is one to one if and only if $\text{Ker}(f) = \{0\}$.
14. Find the orders of the nonzero elements of $\mathbb{Z}/6\mathbb{Z}$.
15. Find $48^{322} \pmod{25}$.
16. Find all solutions of $x^2 \equiv 1 \pmod{8}$.
17. Show that the three vectors $v_1 = 3i - j + 2k$, $v_2 = i + j - k$, $v_3 = i - 5j - 4k$ are mutually orthogonal.
18. Find the area of the parallelogram that has $u = i + j + k$ and $v = i + j$ as adjacent sides.
19. Find $u \cdot (v \times w)$ where $u = 2i - 3j + k$, $v = 4i + j - 3k$, $w = j + 5k$.
20. Check whether the lines L1 and L2 are parallel. Also investigate whether they intersect.
 $L1 : x = 1 + 4t, y = 5 - 4t, z = -1 + 5t$
 $L2 : x = 2 + 8t, y = 4 - 3t, z = 5 + t$
21. Find the distance between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z + 1 = 0$.
22. Describe the surface: $z = -(x^2 + y^2)$.

(8 × 2 = 16 Marks)

SECTION – C

Short essay type problems : Answer any SIX questions.

23. Prove that $\mathbb{Z}/m\mathbb{Z}$ is a field if and only if m is a prime.
24. Let $f : R \rightarrow S$ be a homomorphism where R is a field and $1 \neq 0$ in S . Then show that f is one – to – one.
25. In $\mathbb{Z}/24\mathbb{Z}$ find the inverses of all non – zero elements for which it exists.
26. Give a proof to show that the identity function is the only homomorphism from \mathbb{Q} to \mathbb{Q} , with details.
27. Find the orthogonal projection of $v = i + j + k$ on $b = 2i + 2j$, and then find the vector component of v orthogonal to b .
28. A force of $F = 4i - 6j + k$ Newtons is applied to a point that moves a distance of 15 meters in the direction of the vector $i + j + k$. How much work is done ?
29. Obtain the vector equation of a line passing through $(1, 1, 2)$ and $(1, 2, 3)$.

30. Identify the surface or curve represented by the equations given. Do they intersect ?
If so, find the intersection. $x=t, y=t, z=t; 3x-2y+z-5=0$.
31. Find the equation for the surface that results when $z = x^2 + y^2$ is reflected about the plane $x = z$.

(6 × 4 = 24 Marks)

SECTION – D*Long essay type problems : Answer any TWO questions.*

32. Define an operation on the set \mathbb{N} of natural numbers (> 0) by $a * b =$ the least common multiple of a and b .
- Show that this operation is associative and commutative.
 - Find an identity element for \mathbb{N} under this operation.
 - Which elements of \mathbb{N} have inverses ?
 - When is it possible to solve the equation $a * x = b$?
 - If $a * b = a * c$, does it follow that $b = c$?
33.
 - State and prove the Chinese Remainder Theorem.
 - Find all solutions of $11x \equiv 13 \pmod{16}$.
 $13x \equiv 9 \pmod{28}$
34.
 - Discuss the geometric properties of the scalar triple product.
 - Obtain a formula to calculate the distance from a point to a plane.
35. Consider the hyperbolic paraboloid $z = y^2 - x^2$.
- Find an equation of the hyperbolic trace in the plane $z = 4$.
 - Find the vertices of the hyperbola in part (i).
 - Find the foci of the hyperbola in part (i).
 - Describe the orientation of the focal axis of the hyperbola in part (i) relative to the coordinate axes.

(2 × 15 = 30 Marks)
