MAR IVANIOS COLLEGE (AUTONOMOUS)
THIRUVANANTHAPURAM

Reg. No. :………………… **Name :…………………….**

Third Semester B.Sc. Degree Examination, November 2016

First Degree Programme under CBCSS

Core Course: Mathematics – II

AUMM341: Algebra and Calculus I

Time: **3** Hours Max. Marks: **80**

SECTION – A

*Answer all the **TEN** questions. Each question carries 1 mark.*

1. Write all the units of $\langle \mathbb{Q}, +, . \rangle$
2. Give an example of a non–commutative ring.
3. Write the exponent of U_6 ?
4. Define a field.
5. Write the terminal point of **v** = 3i – 2j, if the initial point is (2, –1) ?
6. Write the direction cosines of the diagonal with vertices (0, 0, 0) and (a, a, a) of a cube having side of length a ?
7. Write the parametric coordinate of circular helix in 3–space ?
8. Write the natural domain of $(t) = (\cos\pi t, -\ln t, \sqrt{t-4})$?
9. Find the vector orthogonal to **a** = (0, 1, – 2) and **b** = (4, 0, –3).
10. Find the rectangular coordinate equation of the curve $x = at, y = \frac{a}{t}$.

(10 × 1 = 10 Marks)

SECTION – B

*Answer any **EIGHT** questions. Each question carries 2 marks.*

11. State Chinese Remainder theorem.
12. Find all solutions of $x^2 \equiv 1 \ mod\ (35)$.

P.T.O.

13. 'A field has no zero divisors'. Justify your conclusion.
14. If p is a prime number, prove that $\mathbb{Z}/p\mathbb{Z}$ is a field. Is the converse true ?
15. Write the unit vector in 2–space that makes an angle of $\frac{\pi}{6}$ radian with the positive x–axis.
16. If $\boldsymbol{u}$ and $\boldsymbol{v}$ are two non–zero vectors in 3–space, prove that $\|\boldsymbol{u}+\boldsymbol{v}\|^2 + \|\boldsymbol{u}-\boldsymbol{v}\|^2 = 2\|\boldsymbol{u}\|^2 + 2\|\boldsymbol{v}\|^2$.
17. Show that the lines $L_1 : x = 3 - t, y = 1 + 2t, L_2 : x = -1 + 3t, y = 9 - 6t$ are same.
18. Sketch the graph of $\boldsymbol{r}(t) = \cos t\boldsymbol{i} + \sin t\boldsymbol{j} + 2\boldsymbol{k};\ 0 \le t \le \pi$.
19. Find the arc length of $x = 5\cos t, y = 5\sin t, z = 12t; 0 \le t \le \pi$.
20. Determine whether $\mathrm{r}(t) = te^{-t}\boldsymbol{i} + (t^2 - 2t)\boldsymbol{j} + \cos 2\pi t\,\boldsymbol{k}$ is a smooth function of the parameter t.
21. Find the curvature and radius of curvature of a circle with center at the origin and radius a.
22. Describe the nature of the surface $y^2 = x^2 + z^2$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any ***SIX*** *questions. Each question carries 4 marks.*

23. Let n be divisible by s distinct odd primes $p_1, p_2, p_3, \ \ldots\ldots\ p_S$. Prove that there are at least 2^s solutions of $x^2 \equiv 1 mod(n)$.
24. State and Prove Abstract Fermat Theorem for finite abelian group.
25. Define Kernel of a homomorphism map in rings. Also prove that $Ker(f) = \{0\}$ if and only if f is 1 – 1.
26. For a finite commutative ring with identity, prove that every non–zero element is either a unit or a zero divisor.
27. Find the unit vector in the same direction as the vector from the point $A(-1, 0, 2)$ to the point $B(3, 1, 1)$.
28. Find the vector component of $\langle 4,1,7\rangle$ orthogonal to $\langle 2, 3, 2\rangle$.

29. Write the rectangular coordinates of the point with cylindrical coordinates $(4, \frac{\pi}{2}, 1)$.

30. Find the parametric equation of the line tangent to the graph $r(t) = 2\cos\pi t\mathbf{i} + 2\sin\pi t\mathbf{j} + 6t\mathbf{k}$ at $t_0 = \frac{1}{3}$.

31. Show that the arc length of the circular helix $x = a\cos t$; $y = a\sin t$; $z = ct$ for $0 \leq t \leq t_0$ is $t_0\sqrt{a^2 + c^2}$.

(6 × 4 = 24 Marks)

SECTION – D

*Answer any **TWO** questions. Each question carries 15 marks.*

32. i). Write all the elements and its orders of U_{20}, the group of units of $\mathbb{Z}/_{20}\mathbb{Z}$. Also find the exponent of this abelian group.

    ii). Prove that for any $a \in G$, $a^n = e$, for a finite abelian group G of exponent n.

33. i). A force of F = 4i – 6j + k Newton is applied to a point that moves a distance of 15 meters in the direction of the vector i + j + k. How much work is done ?

    ii). Find the volume of the tetrahedron with vertices $P(-1, 2, 0)$, $Q(2, 1, -3)$, $R(1, 0, 1)$ and $S(3, -2, 3)$.

34. i). Find the intersection of the line $x = -1 + 3t$, $y = 5t$, $z = -2 - t$ and plane $2x + y + z = 2$.

    ii). St. Petersburg, Russia is located at 30^0 east longitude and 60^0 north latitude. Find its spherical and rectangular coordinates relative to the coordinate axes using navigation. Take miles as the unit of distance and assume the earth to be a sphere of radius 4000 miles.

35. A shell is fired from ground level with a muzzle speed of 320 ft/s and elevation angle of 30^0. Find

    i). parametric equations for the shell's trajectory.

    ii). the maximum height reached by the shell.

    iii). the horizontal distance travelled by the shell.

    iv) the speed of the shell at the time of impact

(2 × 15 = 30 Marks)

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